The Analysis of Dominance and Bidirectionality in Social Development

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GOTTMAN, JOHN M., and RINGLAND, JAMES T. *The Analysis of Dominance and Bidirectionality in Social Development.* CHILD DEVELOPMENT, 1981, **52**, 393–412. This paper is a reexamination of 2 concepts that have played a role in the direct observation of social interaction and the study of social development, namely, the concepts of dominance and bidirectionality. This paper argues that a reconceptualization based on the sequential character of social interaction would add much theoretical clarity to these 2 concepts. In this paper the sequential nature of social interaction is addressed by applying time-series analysis to *redefine* (as a function of social context) the concept of *dominance* as asymmetry in predictability in the behavior of 2 interacting individuals. The potential benefits of this redefinition are then discussed in the context of literature on children's social behavior with their peers. *Bidirectionality* in social interaction, defined as symmetrical predictability in behavior, is then discussed in the context of parent-infant interaction. A serious methodological problem is then raised, namely, the problem of autocorrelation in each person's behavior in making inferences about cross-correlation between people. Mathematical models and corresponding statistical procedures are presented to solve this problem. Procedures are discussed that address the concepts of *cyclicity* within a person and *synchronicity* between people who are interacting and that assess asymmetry and symmetry (i.e., dominance and bidirectionality) in social interaction. Previously published data on mother-infant play is then reexamined to illustrate the practical use of this approach. These techniques are, however, completely general for inferring relationships between 2 variables that change with time and thus may provide a foundation for the study of other developmental problems.

There is a rapidly growing interest in the study of social development using observational methods, and this interest spans many areas of developmental psychology, for example, ethology (see Wilson 1975) and developmental studies of parent-child interaction (see Stern 1977). There is also an increasing recognition of the inadequacy of frequencies or rates of specific social behaviors for summarizing data on social interaction and, consequently, a new concern for the detection of *patterns over time* and the assessment of varieties of interconnectedness between two interacting individuals.

The study of data over time is ideally suited to time-series analysis, and this paper will argue that considerable theoretical clarity can be obtained using time-series concepts. One objective of the paper is to reconceptualize two concepts that have played central roles in the study of social interaction and social development—dominance and bidirectionality. A second objective of this paper is methodological. Inference about cross-correlation between two variables that change over time must allow for autocorrelation within each series. The general analysis of time-related series has been extensively considered (see Anderson 1971, Box & Jenkins 1970, Jenkins & Watts 1968) and the particular problem of cross-correlation has been of interest to econometrics for at least 10 years (see Pierce & Haugh 1977). This paper presents an approach that combines several such concepts into one integrated method, we will attempt to explain this method with a minimum of mathematics. Derivations, where needed, will be given in Appendix C.

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Dominance

The concept of dominance has been operationalized in many ways and applied to an enormous range of social phenomena In this discussion we refer only to dominance concepts designed to characterize social interaction rather than the traits of individuals Concepts designed to study the former are usually specific to group composition and situational context, and this is the sense in which we are interested in dominance

By *dominance*, researchers always refer to some *asymmetry* usually in the frequency of some specific variable designed to operationalize the dominant behavior For example, in research on family interaction, Hetherington and Martin's (1972) review listed these variables who speaks first, who speaks last, and passive acceptance of a solution on a problem-solving task Jacob (1975) listed talk time, number of statements, number of successful interruptions, and number of all acts Asymmetry on one or more of these variables is always the basis for a decision about dominance

There are several problems with defining dominance as asymmetry in these rates or frequencies First, the definitions assume that an isomorphism exists between dominance and asymmetry in the relative *frequency* of a specific behavior, however, in some families the most dominant member may be a person who does not speak very often, who rarely interrupts, but who has great impact when speaking For example, if a dominant member makes a demand, subsequent compliance by others might be more likely than is the case with less dominant members In this case we would expect the asymmetry to be detectable in *patterns* of interaction rather than in the relative frequency of a specific code, which is a mean over time

A second problem with these definitions of dominance is that dominance may not be detected in one variable but in overall patterns across several variables At first glance this problem may seem reasonably trivial, however, historically, the selection of the appropriate variables has been a serious problem For example, in the ethological literature on the naturalistic observation of animal behavior, dominance has been defined in terms of asymmetry in resource utilization, territory, freedom of mobility, and the successful outcomes of aggressive bouts One promising variable for assessing dominance would appear to be the outcome of aggressive bouts over competition for the utilization of resources However, in naturalistic observation of intact social groups, these events have been difficult to observe for several reasons First, one of the functions of a dominance structure is to minimize aggressive encounters, and they are thus infrequent Second, as Wilson (1975) noted "Serious difficulties in the dominance concept appeared as soon as the idea was extended to the more complex life of primates

Some writers then recognized that in both primates and wolves a rich repertory of signals is used to denote status in a manner not directly coupled with aggressive interactions Status signs were seen to be metacommunicative, indicating to other animals the past history of the displaying individual and its expectation of the outcome of any future confrontations" (p 281) There is also some evidence that highly "telegraphed" dominance behaviors, that is, the abbreviated vestiges of previously more elaborated sequences of actions, are used to remind a subordinate of its status Stephenson (Note 1) found evidence for an idiosyncratic "dominance dialect" in a particular alpha male in a group of rhesus monkeys The alpha male, the most dominant member, had an unusual spastic motion of his arm out to one side When he died, the new alpha male, who had never been observed to display this behavior, began doing so To test the hypothesis that this behavior is associated only with the most dominant animal, Stephenson removed this animal, and the new alpha male also adopted this behavior, as if it were the scepter of command Often, the specific behaviors that represent the "dialect of dominance and submission" are present but are subtle and difficult to detect For example, Shirek-Ellefson (cited in Chevalier-Skolnikoff 1973) noted that the threat displays of *Macaca fascicularis* are open-mouthed stares if the animal is dominant and bared-teeth stares if the animal is subordinate

Attempts to extend the notion of a dominance hierarchy to human groups revealed that dominance was even more complex for human than for primate groups For example, it is possible to estimate hierarchies using asymmetry in the outcomes of aggressive encounters of preschool children (McGrew 1972, Strayer & Strayer 1976) by employing linear models (with varying degrees of success), but as Vaughn and Waters (1978) pointed out, "Dissatisfaction with the unidimensional conceptualization of social organization stems primarily from the fact that 'dominance' rankings based on the outcomes of aggressive encounters have not proved very revealing with respect to other aspects of behavior" (p 360) Vaughn and Waters's study of one nursery school class found that linear models of a dominance hierarchy based on two different types of aggressive social encounters—(1) gazing and hostility and (2) object struggles—were not significantly correlated, $r = .20$

We wish to propose that an alternative to searching for asymmetry in the rates or relative frequencies of specific behaviors that are assumed to reflect dominance is to operationalize the concept of *dominance* as *asymmetry in predictability*, that is, if B's behavior is more predictable from A's past than conversely, A is said to be dominant This is a definition that could span a range of behaviors, and asymmetry in the predictability of these behaviors could reflect dominance patterns even if the behaviors themselves do not The behavior of the alpha male should be less predictable from the behavior of the beta male than conversely, in general, it is the beta male who must attend to and respond to the behavior of the alpha male, and not conversely

Recent theorizing on dominance supports our proposed definition of dominance An attempt was made recently to extend the dominance concept beyond the arena of aggressive interactions Chance suggested that the asymmetrical patterning of attention, usually assessed by visual gaze, could be used as an index of dominance even in nonaggressive encounters (Chance 1967, Chance & Jolly 1970, Chance & Larsen 1976) This suggestion is entirely consistent with the asymmetry-in-predictability definition, although the predictability definition is more general, for example, the measure could be excitement or activity levels during play as well as gaze

However, there is an important limitation of this equation of asymmetry in predictability with dominance For example, in caretaking interactions, the fact that a mother's behavior is more predictable from her baby's behavior than conversely may be purely a function of the caretaking *context*, in play interactions the same asymmetry may be indicative of the social/cognitive developmental level of the infant If a mother waits for her baby to be interested in play and her behavior is thus highly predictable from her baby's, but not vice versa, it does not make much sense to refer to the baby's behavior as "dominant" in the same sense as an alpha male monkey's aggressive behavior toward a beta male may be called dominant Another example may strengthen this point Vaughn and Waters (1978) found that among preschool children, visual gaze data taken during free play indoors did not correlate significantly with data on aggressive social encounters, although the gaze data were highly correlated with peer picture sociometric data on liking, which measure a child's preferences in affiliation with peers in that social group In other words, children tended to look most at those children with whom they would *prefer* to play but with whom they usually did not play In short, asymmetry in predictability need not always imply dominance Dominance is a function of the interactants and of the situational context of the interaction The conceptual label given to asymmetry in predictability should thus vary as a function of the dependent measures, the context (e g , the goals and tasks of the interaction), and the nature of the interactants We can distinguish four major contexts that have been studied in social interaction (*a*) caretaking (e g , mother-infant feeding), (*b*) play (e g , mother-infant play, peer play), (*c*) competition (e g , object struggles among preschool children), and (*d*) cooperation (e g , decision making or problem solving in families) If an observational study samples from different social contexts, one need not predict a priori that the same dimension would be tapped by asymmetry in predictability, even if the same variable (such as gaze) were used

While dominance always refers to an asymmetry, symmetry in predictability may also be theoretically interesting For example, it may be profitable to identify that point in a child's development when its interaction with its mother becomes bidirectional The bidirectionality implies that the mother is responding to her baby and that the baby is responding to the mother In fact, in the developmental literature there has been lively discussion of bidirectionality and we will review this discussion

Bidirectionality

Bell's (1968) paper on the bidirectionality of effects in research on child rearing led researchers of parent-infant interaction to begin viewing their observational data as bidirectional This changing view of the infant's capability was made possible by research in the 1960s that found the infant's repertoire much larger and more sophisticated than had been expected (e g , Fantz 1964) To understand the nature of this breakthrough we need to recall that the historical context for the renewed study of parent-infant interaction in the late 1960s was personality theory and that as late as 1965 Handel wrote a review paper suggesting that the central developmental issue in this area was to understand to what extent the child's personality is predictable as some linear combination of the parents' personalities The shift to a model of an active infant was a shift toward a systems view of development, it was not just that the infant was not a *tabula rasa*, but the infant had a piece of chalk of its own The shift also included the movement toward including the study of psychological processes (e g , social interaction) as well as developmental outcomes (e g , personality and cognitive levels)

The developmental issue is the nature of bidirectional social interaction and its implications for the infant's developing social competence To understand this point, consider the fact that it is apparently irresistible for researchers not to speak of the infant's behavior as intentional, planful, and goal oriented For example, Stern (1977) wrote, "Beginning around the third month, the smile takes another developmental leap and becomes an instrumental behavior By instrumental we mean simply that the infant will now produce the smile *in order to get* a response from someone, such as a return smile from mother or a word from her The smile itself, however, still looks the same" (p 45, emphasis added) We suggest that what may have changed in the infant's smile by 3 months is neither the smile nor the mother's response to the smile but the ability to predict when the baby will smile from the mother's behavior The developmental leap may thus be the occurrence of *bidirectionality in predictability*

Bidirectionality has also been studied in pioneering research by Brazelton and his colleagues (e g , Brazelton, Koslowski, & Main 1974, Tronick, Als, & Brazelton 1977) In this research the behaviors of mother and infant are recorded by two cameras and merged on a split screen with a time code that makes it possible to code a 5-min play session in slow motion or frame by frame This detailed quantitative analysis of nonverbal behavior was a major breakthrough in this area An additional breakthrough was the univariate scaling of the microcodes on a dimension of affective involvement The detailed microcodes were categorized into "monadic phases" that were then scaled on a dimension that ranged from maximum negative involvement to maximum positive involvement and excitement This scaling made it possible to examine overall patterns in interaction that might not have been tapped by one isolated code, and it simultaneously gave specific behavioral referents to the scale (see Brazelton et al 1974) Tronick et al (1977) observed three mother-infant dyads and summarized their analyses by the time-series graphs displayed as figure 1

The objectives of subsequent analysis of the Tronick et al (1977) data were (1) the detection of cyclicity in both mother's and infant's affective involvement, (2) the detection of synchronicity in a dyad's behavior, and (3) the detection of bidirectionality To accomplish these objectives, Tronick et al (1977) computed correlations between mother and infant scaled scores, each correlation was based on 10 sec of interaction, the first correlation was based on seconds 1–10, the second correlation on seconds 2–11, and so on (see fig 2) They wrote, "High positive correlations would indicate that the mother and the infant were synchronously moving in the same affective direction, high negative correlations that they were changing their affective involvement in opposite directions, and low correlations, either positive or negative, that there was a general lack of relationship or lack of cycling between the changes of their affective involvement" (Tronick et al , p 77) Tronick et al (1977) referred to positive correlations as "synchrony" and conclude that "the analysis of the scaled sum scores shows that the infants are capable of modifying their affective and attentional displays in a reciprocally coordinated manner" (p 78) Furthermore, "the infant is able to communicate his or her intent and to respond to the expressed intent of the mother He or she can sequence expressive displays in an appropriate serial order while allowing for the turn taking necessary to the reciprocal exchange of messages" (p 79) To summarize, Tronick et al were thus interested in making the following conclusions (1) that mother and infant play is synchronous and reciprocal and (2) that the infant is responding to the mother, that is, that effects are bidirectional From these conclusions they hoped to construct models of the infant's social competence

Unfortunately, it is not possible to support these conclusions from the running correlations If the mother and infant's behavior were entirely cyclic, with unequal frequencies, the behavior of each would be completely determined by its own past and no knowledge would be gained by knowing the behavior of the other

This is a case of no interaction Nonetheless, the Tronick et al running correlations would be nonzero The mother's and baby's cycles would be successively in phase (giving positive correlations) and out of phase (giving negative correlations) This fact is not difficult to prove mathematically [1]

One might, at first glance, suppose that the problem is that Tronick et al only examine some of their cross correlations, that is, only those at lag zero, and only for part of the data at a time However, this is not the core of the problem in inferring bidirectionality What needs to be demonstrated is more than that

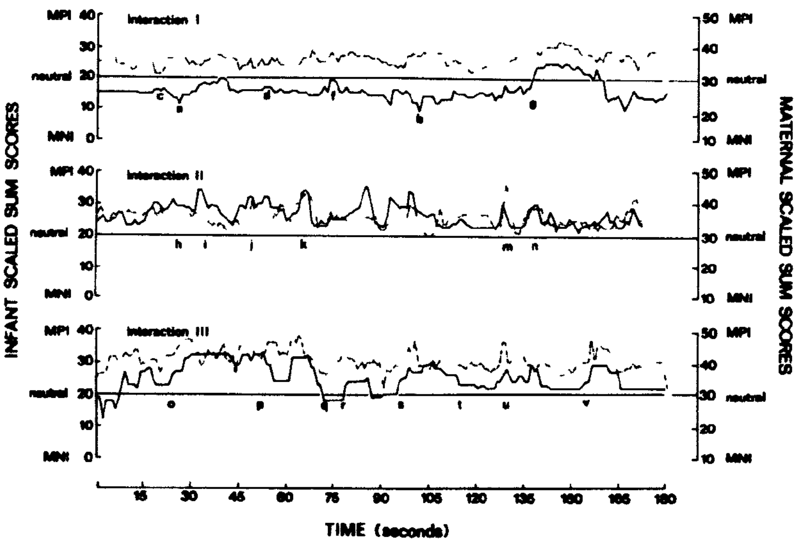

FIG 1 —Scaled sum scores for each of the infant-mother dyads The scores range from maximal positive involvement in the interaction through neutral to maximal negative involvement in the interaction for the infant and mother

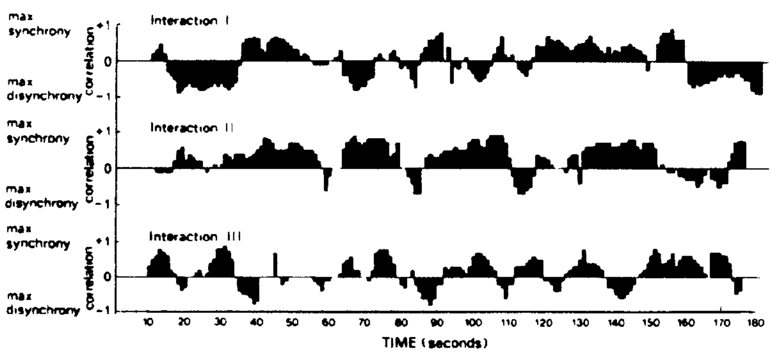

FIG 2 —"Ten second 'running' correlations of infant and mother scaled sum scores Each bar is the correlation of ten seconds of scaled scores for infant and mother The correlation moves in 1 second jumps Thus, the first correlation is for seconds 1–10, the second for 2–11, the third for 3–12, and so on A high positive correlation reflects maximum synchrony in the interaction and a high negative correlation represents maximum disynchrony" (from Tronick et al 1977)

[1] If the mother's behavior cycles at frequency M and the baby's at frequency B, the Tronick running correlations can be written out mathematically and shown to be cyclic with several "modulated" cyclicities, depending on the mother's and baby's cycles Different patterns in the

predictability (at any lag) exists from mother to infant and from infant to mother By itself this demonstration is inadequate to infer bidirectionality because of the problem of autocorrelation This was recently pointed out by Sackett (1980), who wrote, 'The basic issue of autocontingency has not been addressed by students of social interaction Unfortunately autocontingency does affect the degree to which crosslag dependencies can occur In some instances, apparent cross contingencies may be a total artifact of strong autolag functions' (p 330) Sackett's point is consistent with Jenkins and Watts (1968), who showed that "very large cross covariances, all of them spurious can be generated *between* two uncorrelated processes as a result of the large autocovariances *within* the two processes" (p 338)

To demonstrate that the behavior of the baby is influenced by the behavior of the mother, this paper suggests that *we need to show that we can reduce uncertainty in the infant's behavior from our knowledge of the mother's past behavior, over and above our ability to predict simply from the infant's past* A similar discussion can be found in a recent paper by Pierce and Haugh (1977) on assessing causal relationships between economic time series Bidirectionality occurs when we can demonstrate the converse as well, and asymmetry in predictability occurs when we cannot demonstrate this symmetry

We will now discuss the techniques of time-series analysis that address each of the objectives of the Tronick et al (1977) paper We begin with a discussion of stationarity, of the need for describing the data in such a way that the same patterns of relationship hold throughout the series, we continue with the detection of cyclicity and synchronicity, and, finally, we develop the mathematical models and significance tests for assessing bidirectionality The mathematical discussion of the mother-infant data presented in the remainder of this paper is completely general and applicable to other types of social interaction However, as we have pointed out the conceptual interpretation of the statistics may vary with interactional context These analytic methods are currently being applied in our laboratory to affective expression in marital interaction, in which asymmetry is best interpreted as differences in emotional responsiveness, and to asymmetry in gaze probabilities in sibling interaction, in which asymmetry is interpretable as dominance To conserve space, we will not present data taken from these interactional contexts but will limit our discussion to mother-infant interaction

Detecting and Correcting for Nonstationarity

Time-series analysis assumes that the data are *stationary*, that is, that the data vary about a fixed mean and the same pattern of auto- and cross-covariance holds independent of historical time Moreover, time-series analysis, like any other statistical analysis, requires a certain amount of replication and repetition Thus, for the data to be usefully analyzed, the length of the series should be considerably longer than duration of the important auto- and cross-correlational effects being analyzed However the data may not be stationary, and, indeed, the presence of nonstationarity may be one of the most interesting facets of a set of data At the simplest level there may be general trends—linear, seasonal, or otherwise—in the data There may also be local trends—up for awhile, then down, then perhaps level Patterns of correlation may change, especially at the beginning of a record as patterns just begin to assert themselves, or at the end as they decay Or there may be some constant change in autocorrelation and cross-correlation through time

When the original data are nonstationary, several alternatives exist, we will review two (see Chatfield [1975] for another) First, it may be possible to divide the data into successive chunks, such that the data are reasonably stationary within each chunk Second, the data may be transformed so that the transformed data are stationary Box and Jenkins (1970) suggested that the difference from observation

running correlations could be obtained by suitably modifying *B* and *M* Nonetheless, the mother and baby in this example could not be called interactive, their nonzero correlations are purely an artifact of their cyclicity By modulated cyclicities, the following is implied The sum of two sine waves with frequencies *G* and *H* can also be written as the product of two times a sine wave with frequency $(\frac{1}{2})(G+H)$ and a cosine wave with frequency $(\frac{1}{2})(G-H)$ This means that the sum is not a simple sine wave but "modulated" as follows It has a new frequency, which is the average of those in the sum, and a nonconstant amplitude that varies rhythmically with slower frequency $[(\frac{1}{2})(G-H)]$ These rhythmic changes in amplitude are called *beats* For an illustration, see Courant and John (1965, p 581) to observation (the slope), or perhaps the difference of the differences (the acceleration), may be stationary. The differencing transformation creates a new time series, y_t, from the original time series, x_t, by the transformation $y_t = x_t - x_{t-1}$, so as to produce $y_2 = x_2 - x_1$, $y_3 = x_3 - x_2$, $y_4 = x_4 - x_3$, and so forth. Thus, in any time-series analysis we must first examine the data carefully and either divide the data into separate stationary pieces or, to the extent necessary, transform the data so that the transformed data are stationary.

The issue of nonstationarity is complex, and a full discussion is not possible in the space of this paper (the reader is referred to Box & Jenkins [1970] and Chatfield [1975]). However we can note here that the autocorrelations cannot be consistently large for stationarity because this suggests that long-term polynomial trends exist in the data which will necessarily make the autocorrelations insensitive to local temporal variation. The long-term trend can be removed by differencing an appropriate number of times (Box & Jenkins 1970) or by estimating the polynomial trend and subtracting it out (Chatfield 1975).

Detection of some forms of nonstationarity is rather easy. The estimated autocovariances of a stationary time series die out rapidly,[2] hence, as a rule of thumb, if the autocovariances show any consistent pattern for lags over $T/6$, where T is the number of observations, nonstationarity is suggested. The figure $T/6$ is a rule of thumb proposed by Box and Jenkins (1970). The rationale is that estimates of autocorrelations for large lags are less reliable than estimates for short lags because they employ increasingly fewer data points, for lag k only $T - k$ data points are used. It should be noted that to detect one stochastic cycle of any period only two lags are needed, not the number of lags equal to the period. Hence, extremely complex combinations of cyclicity can be detected with $T/6$ lags, if T is reasonably large. If consistent patterns are detected in the autocovariances, differences should be considered, the autocovariance function of the series of differences examined, and, if necessary, differences should be taken again. Differences will simultaneously eliminate nonstationarity due to trends, strictly seasonal low-frequency components, and nonstationarity in cyclicity in the low-frequency ranges.

From the Tronick et al. (1977) data, a first differences transformation was necessary to achieve stationarity.[3] As a representative example, the autocovariance functions for the mother of dyad 3 are shown in figure 3*A* and *B* for both the detrended original data and for the series of differences. Note the patterned large autocovariances for lags beyond 20 in the original data but the smaller and rather random autocovariances for lags greater than 10 in the series of differences.

The original discussion of the data by Tronick et al. also suggests that differencing is appropriate. In examining the running correlations for synchronicity, Tronick considered agreement not between the affective levels of mother and baby but, rather, agreement in *how they were moving* or, equivalently, how well the *differences* between successive observations agreed. Thus, considering differences is a direct extension of the original Tronick et al. (1977) analysis although we also consider autocorrelation and cross-correlation at larger lags.

Detecting Cyclicity and Synchronicity

It is not uncommon for researchers who study social interaction to comment on the periodicity and cyclicity of social behavior and to sense intuitively the need for analytic methods to detect these patterns. In fact, cycles and synchronicity are particularly interesting to researchers of parent-infant interaction. However,

[2] To detect nonstationarity, the speed with which the autocorrelations, r_k, die out (decrease to zero) with increasing lag, k, can be used. A useful rule of thumb is that r_k^2 should decrease faster than $1/k^a$, where $a > 1$. Periodic increases in r_k suggest a nonstationary *deterministic* cycle, which should be removed from the data (see Chatfield 1975).

[3] The Tronick et al. data were, on closer examination, nonstationary for a reason not suggested in the general discussion. Examination of the spectral density function (see p. xx) computed for the first and second halves of the data indicated that, as the play progressed, the cyclicity became faster. This result is consistent with observations made by Daniel Stern (Note 2) that as the play progresses, the infant's and the mother's tempo increases. Because the greatest difference between the two halves of the data were in the slowest cyclicities, after differencing, this effect was not very large. Nonetheless, this check for nonstationarity revealed some new information about the data. Thus, a general recommendation is that, although transformations can eliminate many forms of nonstationarity, it can be extremely informative to model the nonstationarity so that the precise work performed by the transformation is revealed.

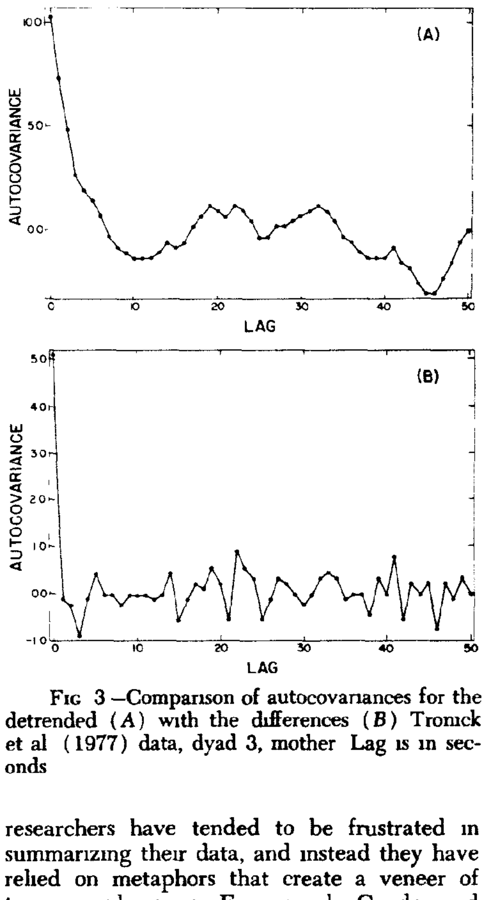

FIG. 3.—Comparison of autocovariances for the detrended (*A*) with the differences (*B*) Tronick et al. (1977) data, dyad 3, mother. Lag is in seconds.

researchers have tended to be frustrated in summarizing their data, and instead they have relied on metaphors that create a veneer of time-series language. For example, Condon and Ogston (1967), using slow-motion film, undertook the frame-by-frame microanalysis of 15 min of the dinnertime interaction of a father, mother, and son. They wrote, "We are dealing with ordered patterns of change during change, which exhibit rhythmic and varying patterns in the temporal sequencing of such changes. Metaphorically, there are waves within waves within waves, with complex yet determinable relationships between peaks and the troughs of the levels of waves, which serve to express organized processes with continually changing relationships" (p. 224). This bewildering array of metaphors is an excellent intuitive prologue to spectral time-series analysis methods.

In a recent paper, Gottman (1979) reviewed the methods of spectral time-series analysis for detecting cyclicity in social interaction. The reader is referred to that paper for details on analyzing time-series data for cyclicity and synchronicity in social interaction. We completed a spectral analysis of the Tronick data and recommend such an analysis for estimating the initial values for the number of parameters necessary in the models we will recommend in this paper. The spectral analysis of the Tronick data is useful for increasing the efficiency of the time-series analyses but, technically, is not absolutely necessary. To conserve space, and to simplify our discussion, we present these spectral analyses in Appendix B. By themselves the spectral analyses are inadequate to test for dominance and bidirectionality because autocorrelation must be controlled in inferring cross-correlation.

The Analysis of Bidirectionality

To augment the frequency domain analyses, it is essential to construct mathematical models in the time domain that control for autocorrelation in the data in the assessment of cross-correlation. Fortunately, there is a precedent of model building that we can call upon in the time-series literature. We will apply the transfer function models proposed by Box and Jenkins (1970). If the mother's time-series in figure 1 is denoted M_t and the baby's series by B_t, we will wish to write a model in which we first try to account for as much of the variance in each series by knowledge of its past and then determine how much we can improve this relationship by adding knowledge of the past of the other series. Stated mathematically, this model is

$$M_t = \sum_{i=1}^{A} a_i M_{t-i} + \sum_{i=1}^{B} b_i B_{t-i} + e_t , \quad (1)$$

and

$$B_t = \sum_{i=1}^{C} c_i B_{t-i} + \sum_{i=1}^{D} d_i M_{t-i} + n_t$$
$$t = 1, 2, \ldots, T , \quad (2)$$

where we assume that e_t and n_t are independent, normally distributed with means zero and variances σ_e^2 and σ_n^2, respectively. We will develop a maximum-likelihood procedure for testing the bidirectionality hypothesis. After a conceptual discussion in the main text, we shall present an example of the actual analysis, with the necessary SPSS programs, to illustrate the implementation of these ideas.

To summarize the mathematics in English, one way of thinking of the model is to transfer the first summation on the right-hand side of each equation to the left-hand side. The model thus attempts to *predict the residual from an autoregression on each series from the past of the other series* The innovation processes e_t and n_t are the residuals of this prediction If there is no gain in this prediction, this means that a smaller autoregressive model without the past of the other series is adequate We thus seek tests for comparing smaller models with bigger models Because of autocorrelation we will not be able to derive exact *F*-ratio statistics, but we will be able to develop likelihood ratio tests with asymptotic distribution theory These tests are generalizations of the univariate procedure discussed in Anderson (1971) Pierce and Haugh (1977) discuss this model, although their analysis differs slightly

There may appear to be an indeterminacy in the model The mother's behavior depends on her own past as well as on the baby's past, which in turn also depends on the mother's past However, because of the assumption that the mother and baby are introducing independent innovations e_t and n_t, there are unique estimates for all parameters Moreover, the estimates of the a_i's and b_i's do not depend on the c_i's or d_i's, so equations (1) and (2) can be handled separately The proof of this assertion is available on request from the first author

We will apply a least-squares procedure originally proposed by Mann and Wald (1943) for estimating the a_i and b_i for a given A and B conditional on "start up" observations M_0, M_{-1}, , M_{-A+1} and B_0, , B_{-B+1} The first A or B observations (whichever is larger) at the beginning of the play sessions are thus treated differently from those once the session is established The parameter vector for the a_i's and b_i's is estimated by ordinary least-squares regression [4] The residual mean square error σ_e^2 will be estimated in the usual way Details are given in the Appendices to this paper

We will fit a model with A and B larger than necessary and test at the 10% level whether $a_A = 0$ or $b_B = 0$ Note that this is not the usual hypothesis-testing situation We wish to include terms if there is *any* evidence at all that they are nonzero Thus, we use the analysis informally, we choose the alpha level to be rather large and do not worry about multiple comparison problems If $a_A = 0$ or $b_B = 0$ we fit a smaller model and repeat the testing and thus step-by-step reduce the model to appropriate size to find the best full model to describe the series

Employing the least-squares estimation procedure *reduced* "null hypothesis" models for the mother's and the baby's behaviors, which assume no interaction can also be fitted In this case we have a reduced model in which the only terms we consider are the past of each series, terms from the other series are dropped out Stated mathematically, this is

$$M_t = \sum_{i=1}^{A^*} a_i{}^* M_{t-i} + e_t{}^* , \qquad (3)$$

and

$$B_t = \sum_{i=1}^{C^*} c_i{}^* B_{t-i} + n_t{}^* \qquad (4)$$

A new pair of estimates for the variances of $e_t{}^*$ and $n_t{}^*$, namely, $(\hat{\sigma}_e{}^*)^2$ and $(\hat{\sigma}_n{}^*)^2$, respectively, can be found To test whether the cross-regressive terms in the final models of the form (1) and (2) significantly help predict the mother's or baby's behavior, we will compare model (1) with (3), using the same value for A and A^*, and model (2) with (4), using the same value for C and C^*, with the likelihood ratio test procedure described below This is the more usual testing situation, and we will require significance at the 5% level before asserting that the cross terms do contribute, that one series is partially predictable from the other Figure 4 gives a summary of the four regions that are possible outcomes of these tests

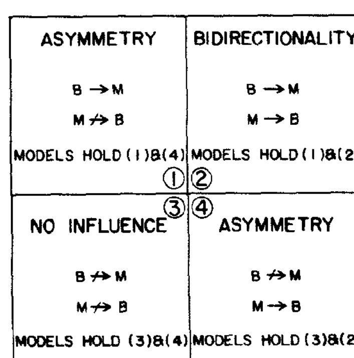

FIG 4 —Four quadrants illustrating bidirectionality, dominance (or asymmetry in predictability), and no interaction for two individuals, *M* (mother) and *B* (infant)

[4] Differencing can be suggested by the least-squares autoregressive parameters, e g, if $a_1 = 1\ 0$, or is close to 1 0, first differencing may be useful

If $B \rightarrow M$ denotes that the mother's behavior is predictable from her infant's, then region 2 represents bidirectionality, and regions 1 and 4 represent asymmetry In another context these two regions could represent dominance patterning

In addition, we will want to compare models of the form (1) and (2) with different A, B, C, and D to check whether, overall, the initial term-reduction procedure was justified and to compare models of the form (3) or (4) with different A^* and C^* to check whether additional terms in a purely autoregressive model give a significantly better fit Neither of these two tests should give significant results They are internal checks on the validity of the procedure

Each of these model comparisons is carried out using the following likelihood, equal to ½ ln (error sum of squares/T), where T is a number of observations (not the degrees of freedom for error) If we have two models, one a reduction of the other, the reduced model then adequately describes the data, then the statistic, denoted Q, which is $-2T$ times the difference in log likelihoods, has approximately a χ^2 distribution, where the degrees of freedom is the difference in the number of parameters between the full and the reduced model If Q is too large, we have evidence that the larger model is more appropriate than the reduced model We refer the reader to Appendix A for a specific numerical example and to Appendix C for theoretical discussion of this procedure

In all models compared we are assuming implicitly that T is unchanging Our procedure will be to establish the first 10 terms of each series as start-up observations (so time $= 1$ is the eleventh observation) This is not mathematically necessary The same likelihood ratio tests apply with different T's, and we could take only as many start-up observations as needed in any particular model—more for models with more terms, less for shorter models—but we then have some difficulty interpreting results about slightly different stretches of data

For the Tronick et al (1977) data, it was also necessary to truncate the data, tossing out the last observation of each series The last observation was usually uncharacteristic of the pattern in the data, representing the end of the interaction (see fig 1, interactions I and III)

To summarize, we suggest the following time-domain analysis for each of the six series (mother and baby in each dyad) (i) Using spectral estimates (see Gottman 1979), guess appropriate values of A, B, C, D, (ii) starting with slightly larger A, B, C, and D, step by step remove terms to find an appropriate model of the form (1) or (2), (iii) compare the model in ii with the one of the form (3) or (4) where A^* and C^* are the values in the model computed in ii, and (iv) compare the reduced model in iii with one with larger A^* or C^* to check that additional autoregressive terms do not help

The data requirements for the time-series analyses proposed in this paper are (1) data that are stationary or can be made stationary and (2) a stretch of data long enough so that interesting autocorrelational patterns are shorter in duration than the data, which usually would imply at least 150 data points The data need not be normally distributed for the log-likelihood ratio statistics to be distributed asymptotically as described If the data are dichotomous as in the case of occurrence/nonoccurrence of a particular behavior (e g , visual gaze toward other), more than 150 observations are necessary to approach asymptotic conditions whereas if the data are completely normal, the analyses could be safely conducted with fewer observations Note that differencing the data is not a statistical requirement, the analysis requires only two stationary, potentially related time series The methods used in this paper to achieve stationarity were created in response to the data at hand, other methods might be more suitable for other cases We will now give the results of such an analysis on the differenced Tronick data

Bidirectionality Analysis of the Tronick Data

The spectral analyses of the Tronick data provide initial estimates of A, B, C, D for all dyads to be no larger than eight, hence, conservatively, we may start with estimates of $A = B = C = D = 10$ In practice, if the reader does not use spectral analysis, it might be wise to check some of the larger values for A, B, C, and D The results of (i) the original excessively large model with $A = B = C = D = 10$, (ii) the step-by-step reduction, (iii) the model fitted by removing the cross-regressive terms from ii, and (iv) a model with 10 autoregressive terms and no cross-regressive terms are summarized in table 1 For comparison, the residual variance has also been given for the observations without a fitted model The relevant likelihood ratio test results are obtained by comparing (i) the starting model, (ii) the best auto- and cross-regressive model, (iii) the purely autoregressive model, and (iv) the enlarged autoregressive model. Recall that the comparison of i versus ii and iii versus iv should give nonsignificant results, being merely internal checks on our method, while the comparison of ii versus iii will indicate the presence or absence of predictability of one series from the other, controlling for autocorrelation.

In each case, note that the step-down procedure behaved as expected and that adding additional autoregressive terms did not significantly decrease the error variance. For dyad 1, we can assert $B \rightarrow M$, but we do not have sufficient evidence that $M \rightarrow B$. Note that, however, as we have set things up, we assume no cross relation until it is clearly demonstrated.

In dyad 2, we have $B \rightarrow M$ with really no evidence at all that $M \rightarrow B$, that is, no evidence of bidirectionality. This is a case of the mother very closely following an independent baby. Dyad 3 is both more complicated and more interesting. We clearly have $M \rightarrow B$, here for the first time is a baby responding to the mother. In addition, we have evidence at the 5% level that $B \rightarrow M$, that we have indeed a bidirectional dyad. The baby in dyad 3 was the oldest of the three.

Summary

This paper has been a reanalysis of two concepts in the study of social behavior, one involving asymmetry, that is, dominance, and one involving symmetry, that is, bidirectionality. We noted the importance of the context of

TABLE 1

SUMMARY OF THE TIME-DOMAIN ANALYSES

	Mother				Baby			
	A	*B*	*SSE*	*T* ln (*SSE*/*T*)	*C*	*D*	*SSE*	*T* ln (*SSE*/*T*)
				Dyad 1 ($T = 168$)				
i	10	10	212.7	39.62	10	10	345.3	121.02
ii	3	5	228.4	51.61	8	7	357.3	126.75
iii	3	0	249.6	66.50	8	0	380.7	137.45
iv	10	0	240.8	60.48	10	0	372.7	133.85
	0	0	257.0	71.42	0	0	415.0	151.92
i vs ii	$Q(12) = 11.99$, N.S.				$Q(5) = 5.73$, N.S.			
ii vs iii	$Q(5) = 14.89$, $p < .025$				$Q(7) = 10.70$, N.S.			
iii vs iv	$Q(7) = 6.02$, N.S.				$Q(2) = 3.60$, N.S.			
Conclusions	$B \rightarrow M$				$M \nrightarrow B$			
				Dyad 2 ($T = 161$)				
i	10	10	625.9	218.61	10	10	511.6	186.13
ii	8	1	660.2	227.19	10	8	519.4	188.56
iii	8	0	705.3	237.82	10	0	540.1	194.88
iv	10	0	703.1	237.32				
	0	0	834.0	264.81	0	0	694.0	235.23
i vs ii	$Q(11) = 8.58$, N.S.				$Q(2) = 2.43$, N.S.			
ii vs iii	$Q(10) = 10.63$, $p < .001$				$Q(8) = 6.31$, N.S.			
iii vs ii	$Q(2) = .50$, N.S.							
Conclusions	$B \rightarrow M$				$M \nrightarrow B$			
				Dyad 3 ($T = 168$)				
i	10	10	674.9	233.62	10	10	412.1	150.74
ii	4	8	688.8	237.05	4	5	447.4	164.55
iii	4	0	759.3	253.41	4	0	503.4	184.37
iv	10	0	745.2	250.27	10	0	479.1	176.05
	0	0	810.0	264.28	0	0	515.0	188.19
i vs ii	$Q(8) = 3.43$, N.S.				$Q(9) = 13.81$, N.S.			
ii vs iii	$Q(8) = 16.36$, $p < .05$				$Q(5) = 19.82$, $p < .005$			
iii vs iv	$Q(6) = 3.14$, N.S.				$Q(6) = 8.32$, N.S.			
Conclusions	$B \rightarrow M$				$M \rightarrow B$			

the interaction and the nature of the interactants in employing the terms, noted that both terms imply *social influence,* which could be assessed by *predictability* Consistent with this discussion, we suggested that dominance can be defined as asymmetry in predictability in social variables of importance, and bidirectionality as symmetrical predictability We discussed the conceptual advantages of these definitions and the usefulness of a method for quantitative assessment taken directly from the stream of behavior

We applied time-series analysis in the time domain for this assessment and noted that the fundamental assumption of stationarity must first be considered We mentioned the analysis of cyclicity and synchronicity using spectral time-series methods but pointed out the limitations of this analysis when estimating lead-lag relationships to make inferences about bidirectionality because these analyses do not control for autocorrelation We then constructed a time-domain mathematical model (which may be assisted by the spectral analyses) and the statistical tests for assessing whether social influence exists, and if so, which is the particular form of asymmetry or bidirectionality

Appendix A
Sample SPSS Calculations

In this Appendix we detail the computations needed to carry out the proposed analyses The computations were carried out with version 8 0 of SPSS as implemented on the University of Illinois Cyber 175 We shall make use of subprogram SPECTRAL to obtain the autocovariances and optional spectral analyses and then use subprogram REGRESSION for the time-domain analyses If SPECTRAL is unavailable, PEARSON CORR can be used, with a bit more programming difficulty in establishing the lagged variables and without the graphical displays SPECTRAL provides We present the alternative version at the end of this Appendix

The analysis proceeds in the following steps

Step A Examine the auto covariances and, optionally, the spectra for the raw data The code

```
FILE NAME       TRONICK
VARIABLE LIST   MOM BABY
INPUT MEDIUM    CARDS
N OF CASES      179
INPUT FORMAT    FIXED(2F5 0)
SPECTRAL        BIVARIATE = MOM BABY/
                WINDOW = HAMMING(50)
OPTIONS         4,5
STATISTICS      1,3,5
SAVE FILE
FINISH
```

was used to read the raw data for the first dyad and to produce auto- and cross-correlations and spectral analyses Nonstationarity was suggested by the slow decay and nonrandom looking behavior of the autocorrelations and by the spectra which were essentially zero except at very low frequencies

The WINDOW = HAMMING(50) indicates how the spectral analysis is to be carried out The (50) represents the number of lags used, which in general should not exceed ¼ to ⅓ of the number of observations

Step B Take differences if necessary and repeat step A Because the pictures in step A did not appear appropriate, we ran the regression shown at bottom of page below First, we generate a new variable $M1$ which at time (case) t has value M_{t-1}, DMOM then has value $M_t - M_{t-1}$, and we repeat the earlier analysis

This time the autocovariances were small and rather noisy for all but the first few lags, and the spectrum showed considerable structure, both indications of stationarity, so we can proceed to the time-domain analyses Were this not the case, differences could be formed again Note that the spectra can be used to assist in the following step (see Appendix B)

Step C Fit the initial large model 1 We illustrate here only the analysis of a mother's behavior as a function of her and her baby's past We need to generate the dependent variables using the LAG operation repeatedly and carry out the regression, as shown at top of the following page Option 19 fits the regression without an intercept This is appropriate for differenced data which

```
GET FILE        TRONICK
COMPUTE         M1 = LAG(MOM)
COMPUTE         B1 = LAG(BABY)
COMPUTE         DMOM = MOM − M1
COMPUTE         DBABY = BABY − B1
ASSIGN MISSING  ALL(-100)
SPECTRAL        BIVARIATE = DMOM DBABY/
                WINDOW = HAMMING(30)
OPTIONS         4,5
STATISTICS      1,3,5
SAVE FILE
FINISH
```

```
GET FILE          TRONICK
COMPUTE           DM1 = LAG(DMOM)
COMPUTE           DM2 = LAG(DM1)
COMPUTE           DM3 = LAG(DM2)

COMPUTE           DM10 = LAG(DM9)
COMPUTE           DB1 = LAG(DBABY)
COMPUTE           DB2 = LAG(DB1)

COMPUTE           DB10 = LAG(DB9)
ASSIGN MISSING    ALL(-100)
REGRESSION        VARIABLES = DMOM, DBABY, DM1 TO DB10/
                  REGRESSION = DMOM WITH DM1 TO
                    DB10(2)/
                  REGRESSION = DBABY WITH DM1 TO
                    DB10(2)/
OPTIONS           19
SAVE FILE
FINISH
```

should have a zero mean If regression is carried out with the raw data, do not include this option

The resulting output provides the following information First, from the ANOVA table we note that the residual sum of squares (*SSE*) was 212 7 This will be used later for the likelihood ratio tests Second, we use the summary table at the end of the output to see which variables are and are not significant In this case only DM3, DM4, DM6, DB3, and DB5 were significant at the 0 1 level

Note that the way missing data are handled by this subprogram treats the first 11 cases specially since lagged values are missing for these cases DMOM is missing for case 1, DM1 to DM10 are missing for case 2, DM2 to DM10 are missing for case 3, , and DM10 is missing for case 11 However, the values of DMOM for cases 2–11 are used as lagged independent variable values in cases 12–21 Thus *T*, the number of observations, is effectively 179 − 11 = 168 This number can also be obtained as the sum of regression *df*, 20, and the residual *df*, 148

Step D Reduce model i by a backward stepwise procedure We remove the "higher-order" terms a few at a time We noted DM10, DM9, DB10, and DB9 were not significant at the last step, so we try regressing DMOM WITH DM1 TO DM8, DB1 TO DB8, as shown at bottom of page below

The SELECT IF statement guarantees that we use exactly the same cases used in step C Without this statement, this analysis would include two additional cases, making comparisons between the model here and in step C harder to interpret

We note from the summary table that only DM3, DM4, and DB3 are significant (although DB5 is nearly so) Thus on the next step we attempt a regression of DMOM WITH DM1 TO DM6, DB1 TO DB6 This time, only DM3 and DB3 are significant, although DB5 and DM4 are close Next we regress DMOM WITH DM1 TO DM4, DB1 TO DB5 Now we find that DM3, DB3 and DB5 are significant, so we regress DMOM with DM1 TO DM3, DB1 TO DB5 at the next stage Here, both highest-order terms DM3 and DB5 are significant, so we stop the step-down procedure and record the *SSE* 228 4 The test of the validity of this reduction procedure involves the two natural logarithm likelihoods For the original model i we have $T \ln (SSE/T) = 168 \ln (212\ 7/168) = 39\ 62$ and the reduced model ii $T \ln (SSE/T) = 168 \ln (228\ 4/168) = 51\ 61$ The difference, 11 91, is not unusually large when compared with χ^2_{12} critical point Note that 12 terms were removed in the step-down procedure

Step E Testing for directionality We regress DMOM WITH DM1 TO DM3 That is, we remove the baby terms from the final model ii above We note the *SSE* is 249 6, whence the natural logarithm likelihood becomes 65 50 The test statistic comparing this model iii with model ii above gives $Q = 65\ 50 - 51\ 61 = 14\ 89$, which is larger than the $\chi^2_{df\ 025}$ value (We have removed five terms, hence 5 *df*)

Step F Finally, as a check we fit DMOM WITH DM1 TO DM10 and compare the sum of squares from this model iv with model iii to verify

```
GET FILE          TRONICK
SELECT IF         (DM10 NE -100 )
REGRESSION        VARIABLES = DMOM, DM1 TO DM8, DB1
                    TO DB8/
                  REGRESSION = DMOM WITH DM1 TO DM8,
                    DB1 TO DB8(2)/
OPTIONS           19
FINISH
```

that a larger, purely autoregressive model would not significantly improve the fit

If subprogram SPECTRAL is not available, the following, somewhat awkward, construction will produce the autocorrelations for, say, the mother series

```
COMPUTE          M1 = LAG(MOM)
COMPUTE          M2 = LAG(M1)

COMPUTE          M30 = LAG(M29)
PEARSON CORR     MOM WITH M1 TO M30
```

Appendix B
Spectral Analysis

In this Appendix we give a brief overview of spectral analysis, of what information these statistics can supply We then present the results of a spectral analysis of the Tronick data, presenting the additional descriptive information we can gather about the interactions and then showing how this information was used to identify approximately the form of the time-domain models

Spectral Analysis: An Introduction

The concept of "spectral decomposition" of a time series is to approximate the time series as the sum of independent *stochastic* sine waves A stochastic sine wave is one that oscillates at a given frequency, but where the amplitude is a random variable A random frequency can also be expressed as a random amplitude In other words, a stochastic cycle is more or less periodic or "almost periodic" The variance of the whole series is then decomposed as the sum of variances at each frequency Such a decomposition of one series is described as the *spectral density function* The variability contributed by a frequency band (f_1, f_2) is indicated by the area under this curve between f_1 and f_2 Peaks in the spectral density indicate cyclicity They indicate which frequencies most contribute to the series Also, by applying these univariate spectral time-series analyses to the mother's and the baby's time series separately, we can determine if they are oscillating in the same frequency ranges, which is important knowledge for assessing synchronicity

In addition to being valuable descriptive tools, these univariate spectral analyses can shed light on the form of an appropriate autoregressive model For a univariate autoregression, a model with two parameters can approximate a series with one well-defined spectral peak, one of order four can represent two cyclicities, and so on (see Box & Jenkins 1970) Note that this means that only the two lags needed to fit a model of order two and thus to describe a cycle of any period, even one which is much longer than two time units Thus, by examining the spectral densities, we have some indication how many *auto*regressive terms we must include in our models

The spectral decomposition theorem is not obvious In fact, the suggestion in the eighteenth century by Daniel Bernoulli that a wide class of mathematical functions could be expressed as a sum of sines and cosines was rejected by most of the mathematicians of his time (see a historical account by Hawkins [1975]), and it remained for Jean Baptiste Joseph Fourier's (1822/1978) monograph to develop the concept Rigorous work on Fourier's treatise continued for another 120 years, and the work was only recently extended to stochastic series (e g , by Wiener 1933, 1949)

Bivariate spectral time-series analysis provides two additional pieces of information First, the *coherence spectrum*, $\rho^2(f)$, gives the square of the correlation between the random amplitudes for the two series at each frequency Correlation is, however, a measure of association, and no direction of causality can be inferred from it When the coherence is high we have evidence of interaction, but its form must come from other considerations, when it is low we know there is little influence in either direction

Second, the *phase spectrum*, $\phi(f)$, describes the lead-lag relationship at each frequency If two series $x(t)$ and $y(t)$ are considered, with $x(t)$ the input series, then a negative phase indicates that $x(t)$ leads $y(t)$ If $x(t)$ denotes the mother's series, this situation could be interpreted to mean that the mother is leading and the baby is responding to the mother when considering cyclicity at a specific frequency In fact, we can compute, for each frequency component, the baby's response time by dividing the phase $\phi(f)$ by $(-2\pi f)$ We assume here that the phase is given in radians and the frequencies in cycles per unit time (Koopmans 1974, p 95) If the ratio is zero, the series are perfectly in phase and synchronous The phase can be examined at those places in the frequency range where mother and baby are cycling together, that is, where their individual spectral densities peak at the same frequency If the phase spectrum is a straight line, this means that throughout the entire frequency range the same time lead-lag relationship holds A positive slope indicates that the baby leads, a negative slope indicates that the mother leads (for a proof of this latter result see Gottman [1979]) The size of the slope can indicate how many cross-regressive terms are needed in the time-domain models For example, if the time delay is 5 sec, we will need at least five *cross*-regressive terms

Phase information is especially meaningful at those frequencies where the mother and baby are cycling together, that is, where their individual spectral densities peak together and where the coherence is high Indeed, the phase spectrum is interpretable only when the coherence is high Jenkins and Watts (1968) showed that the variance of the sample estimate of the phase is proportional to $(1 - \rho^2)/\rho^2$ Thus if ρ^2 is close to 1 0, the variance of the phase estimate is small, as ρ^2 decreases, the variance increases So, low coherence suggests that any lead-lag relationships are accidental and not indicative of cross-correlational patterns

In practice it is only possible to estimate these spectral parameters for a small set of frequencies (called the "overtone series") Details for such estimation and confidence-interval procedures can be found in Jenkins and Watts (1968) or Koopmans (1974) It should also be noted that these estimation procedures require a rather large amount of data The Tronick series consisted of roughly 175 observations each This was by no means excessive The SPSS programs of Appendix A indicate how to obtain these estimates, although confidence procedures are not available yet

To summarize, we propose the following use of spectral time-series analysis The spectral density estimates should be examined to find which cyclicities dominate the series Regions of high coherence indicate cross-correlation at these frequencies but do not control directly for autocorrelation In such regions the phase spectrum can indicate asymmetric time delays Although these methods do not control specifically for autocorrelation, they assist in building time-domain models which do by suggesting the number of auto- and cross-regressive terms needed

Spectral Analysis of the Tronick Data

In this section we will demonstrate the application of spectral time-series analysis, beginning with the examination of the sample estimates of the spectral density functions for mother and baby and followed by the examination of the estimates of the coherence spectrum and the phase spectrum Following this general description we shall indicate how we used this data to start the time-domain step-down procedure at $A = B = C = D = 10$ Recall that we are examining the differences between observations, so we are considering cyclicity and the synchronicity of the patterns of change in the data, not in the data themselves

We did not perform our analyses using SPSS, so there are a few differences between our plots and those for SPSS First, SPSS labels the spectral intensities by the period $1/f$ instead of the frequency f we show Second, the phase ϕ is given in degrees in SPSS instead of radians, so whereas the vertical scale in figure 5 is from $-\pi$ to π, SPSS gives a range −80 to 180

For dyad 1, note that the spectral densities show the same broad outlines but that the baby tends to be cycling slightly faster For example, the mother's spectrum peaks in the $f = 14$–18 and 36–44 cps (cycles per second) ranges, while the child's spectrum peaks at the slightly higher 20–22 and 42–46 ranges (see fig 6) The coherence is low over both of these ranges, indicating that these similar cyclicities are more a result of autocorrelation than cross-correlation (see fig 5) Only at very low frequencies, which contribute relatively little to the variances of the two series, is the coherence high Nonetheless, examining the phase spectrum suggests that there may be a general linear trend throughout the phase spectrum This is illustrated in figure 5 by the parallel lines The slope of these lines is 30 77, which, when divided by 2π, gives a time lag of 4 90 sec Thus, we can conclude that, although most of the variation in these series involves autocorrelation, there is some indication that at slow cyclicity the two series are interrelated, with the mother responding to the baby at about a 5-sec lag In view of the facts that the coherence is generally low and that the phase spectrum does not control for autocorrelation, this apparent time delay should be considered a hypothesis to be tested rather than a proven interpretation of the data

Figures 7 and 8 illustrate the facts that the spectral densities for the mother and baby for the second dyad both peak in the neighborhood of 10

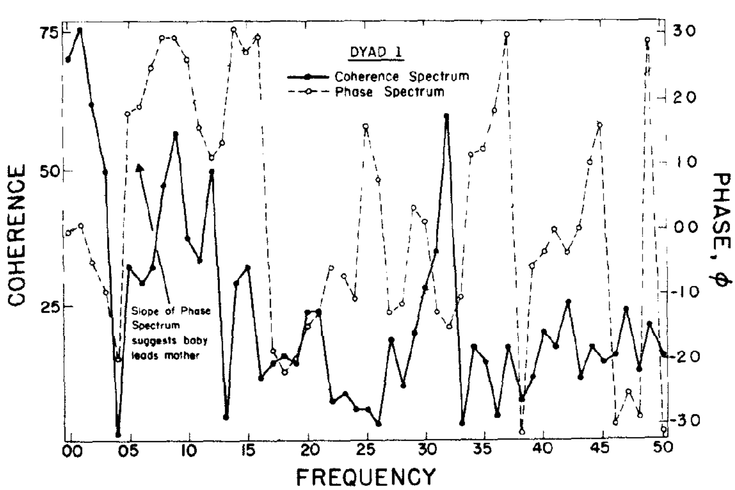

FIG 5 —Coherence and phase (in radians) spectral estimates, dyad 1 (frequency is in cycles per second)

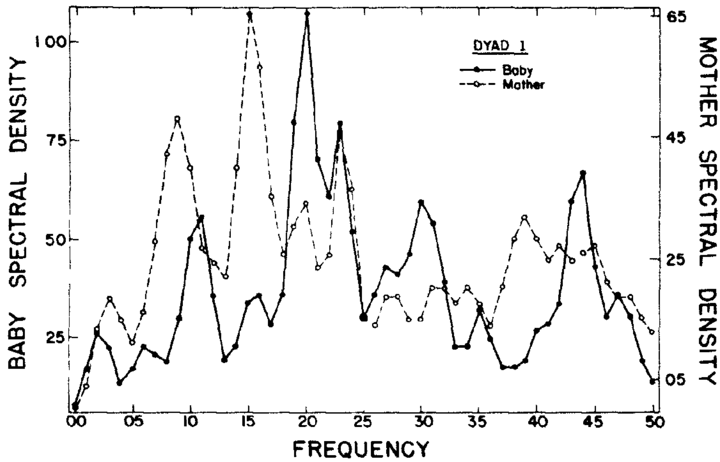

FIG 6 —Spectral density estimates, dyad 1 (frequency is in cycles per second)

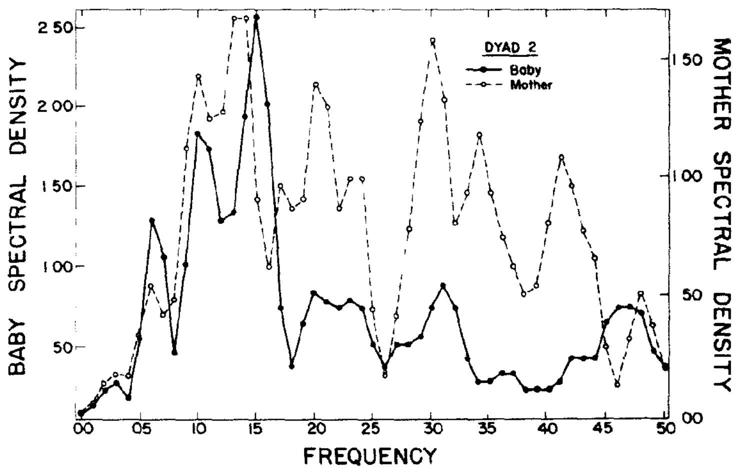

FIG 7 —Spectral density estimates, dyad 2 (frequency is in cycles per second)

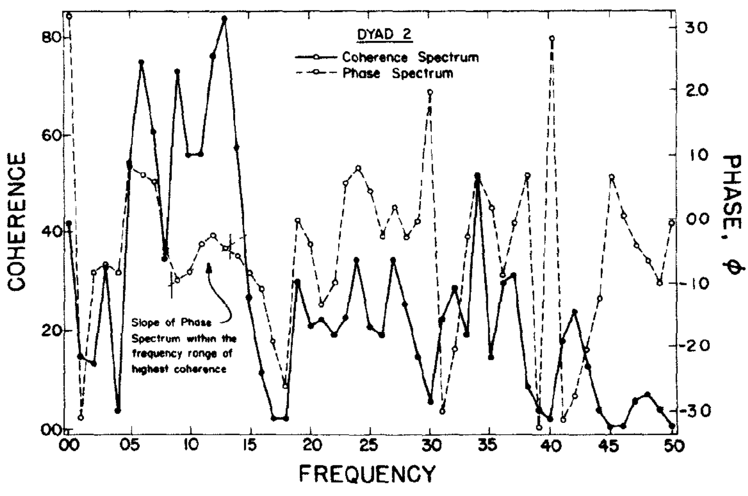

FIG 8 —Coherence and phase (in radians) spectral estimates, dyad 2 (frequency is in cycles per second)

$\leq f \leq$ 15, and this is the frequency range of highest coherence Other peaks appear, but the alignment between the two spectra is less obvious or not present, only at $f = 30$, where the coherence is low, do the spectra peak together again If we examine the phase spectrum in the frequency range of highest coherence, we can conclude that the baby leads the mother, by computing the time delay at the center of this frequency range, we find that the mother is responding to the baby with a time delay of less than 1 sec Thus, for dyad 2 we could hypothesize that there is evidence of strong cross-correlation, but that once again it is the mother who responds to the baby, that is, the direction of influence is asymmetric Once again, we must correct for autocorrelation to test this hypothesis

The third dyad is much more difficult to analyze spectrally The spectral densities do not appear to peak together (see fig 9) The coherence is high near $f = 04$, where the baby is cycling strongly but the mother is not, and near $f = 20$, where the opposite is true (see fig 10) At $f = 04$ the time delay is $-2\ 20$ sec, and at $f = 20$, the time delay is $-0\ 60$ sec, which suggests that the baby follows the mother This is interesting, but we must be cautious in directly interpreting the phase spectrum The frequency $f = 20$ cps corresponds to a cycle with a period of 5 sec Thus a time difference of $-$ 6 sec could also be interpreted as a delay of

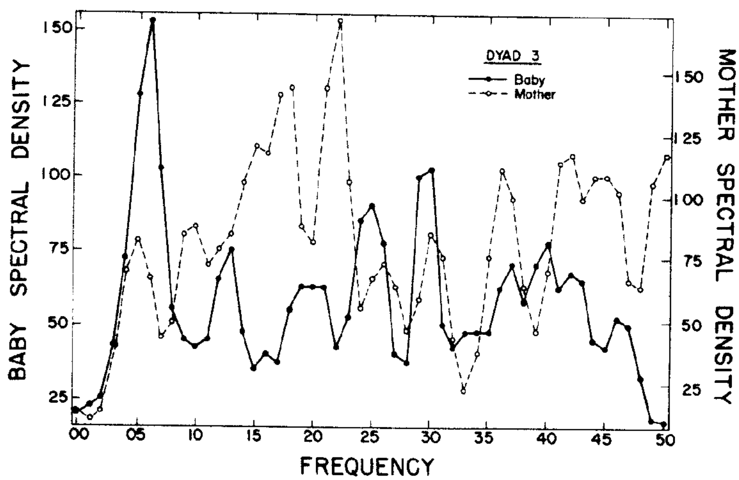

FIG 9—Spectral density estimates, dyad 3 (frequency is in cycles per second)

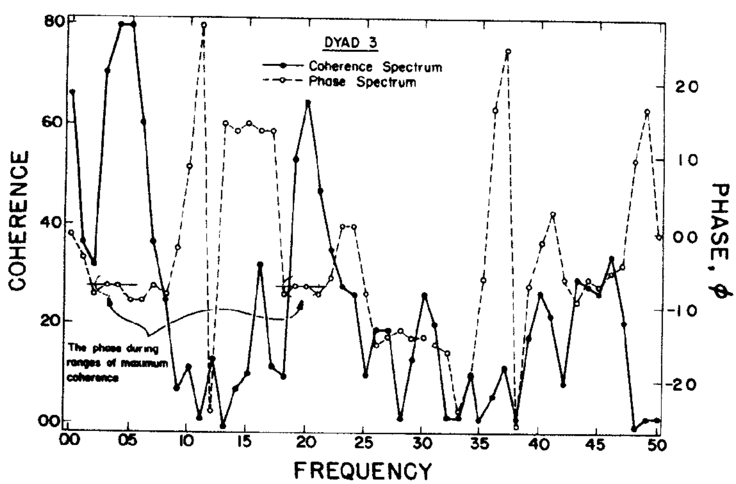

FIG 10—Coherence and phase (in radians) spectral estimates, dyad 3 (frequency is in cycles per second)

$-.6 + 5 = 4.4$ sec or $-.6 - 5 = -5.6$ sec. It is not clear which is appropriate.

With this descriptive insight we now indicate how we can use spectral analyses for time-domain model building.

Examination of the spectral densities suggests the number of autoregressive terms that will be necessary. For dyad 1, the mother's spectrum shows two peaks and the baby's spectrum shows four, suggesting, for this dyad, $A = 4$ and $C = 8$. Similarly, for dyad 2, $A = 8$ and $C = 4$, 6, or 8 appear reasonable. Thus, we will start the step-down procedure at slightly larger values, $A = C = 10$.

Preliminary estimates of B and D are derived from the phase analysis. Recall that in dyad 1 we found the mother responding to the baby with a 5-sec delay. This suggests that B will be at least 5, but it says nothing about D. Indeed, we have no spectral indication about the baby's response to the mother's past behavior. Similarly, for dyad 2, we find that B will be at least 1 with no evidence as to what D may be. Dyad 3 gave ambiguous results for the time delay, since it was not clear whether the partners were synchronous or whether there was a delay, either way, of about 5 sec. Thus we should take B and D to be at least 5. For safety, we started the step-down procedure again at the larger values of $B = D = 10$.

Appendix C
Mathematical Details

In this appendix we relate the procedure described generally in the text to the matrix formula for multiple regression and then develop the likelihood ratio tests we propose.

Matrix Formulation of the Models

The basic model,

$$M_t = \sum_{i=1}^{A} a_i M_{t-i} + \sum_{j=1}^{B} b_j B_{t-j} + e_t ,$$

where $t = 1, 2, \ldots, T$, and (allowing for "start-up observations $M_0, M_{-1}, M_{-2}, \ldots$), can be rewritten in matrix form as $Y = X\theta + E$, where Y is the observation vector, θ the parameter vector, and X the design matrix.

$$Y = \begin{bmatrix} M_1 \\ M_2 \\ \vdots \\ M_T \end{bmatrix} \qquad \theta = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_A \\ \hline b_1 \\ \vdots \\ b_B \end{bmatrix}$$

$$X = \left[\begin{array}{cccc|ccc} M_0 & M_{-1} & \cdots & M_{1-A} & B_0 & \cdots & B_{1-B} \\ M_1 & M_0 & \cdots & M_{2-A} & B_1 & \cdots & B_{2-B} \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ M_{T-2} & M_{T-3} & \cdots & M_{T-1-A} & B_{T-2} & \cdots & B_{T-1-B} \\ M_{T-1} & M_{T-2} & \cdots & M_{T-A} & B_{T-1} & \cdots & B_{T-B} \end{array}\right]$$

which has the familiar least-squares parameter estimates, $\hat{\theta} = (X'X)^{-1} X'Y$ In large samples, these estimates have approximately a normal distribution with mean θ and variance-covariance matrix $\sigma_e^2 (X'X)^{-1}$ where σ_e^2 is the variance of the e_t's

This variance is estimated by the usual mean square error, as shown at bottom of page below For the discussion of the likelihood ratio tests, it will be convenient to introduce the symbol $\hat{\sigma}_e^2$ for the estimate of variance with $1/T$ weighting $\hat{\sigma}_e^2 = SSE/T$

All of this is nearly the same as the usual multiple-regression setting, and, at least in large samples, the same methodology applies

The estimates of the c's and d's for describing the baby's behavior are handled identically and separately, using

$$\begin{bmatrix}\hat{c}\\ \hat{d}\end{bmatrix} = (X'X)^{-1}X'Y ,$$

where in X and Y the roles of M and B are interchanged and C and D replace, respectively, A and B

Likelihood Ratio Tests

The *likelihood function* for a statistical model is the joint probability density of all the random observations, considered as function of unknown parameters If we let $L_1(M,a)$ denote the likelihood function for the smaller model (3) and $L_2(M, B, a, b)$ be the likelihood function for the more general model (1), then assuming normally distributed errors, we can write

$$L_2(M, B, \hat{a}, \hat{b})$$

$$= \frac{1}{(2\pi)^{T/2}} \frac{1}{(\hat{\sigma}_e)^T} \exp - \left[\frac{\Sigma\,(\text{residuals})^2}{2\hat{\sigma}_e^2}\right] \quad \text{(C1)}$$

Noting that $\Sigma(\text{residuals})^2 = T\hat{\sigma}^2$, this reduces to

$$L_2(M, B, \hat{a}, \hat{b}) = (\text{constant}) \frac{1}{(\hat{\sigma}_e)^T} \quad \text{(C2)}$$

or, denoting the natural logarithm of this multiplicative constant by F, $\ln L_2(M, B, \hat{a}, \hat{b}) = F - T \ln \hat{\sigma}_e$ Similarly, with F denoting the same constant,

$$L_1(M, \hat{a}) = F - T \ln \hat{\sigma}_e^* , \quad \text{(C3)}$$

where $\hat{\sigma}_e^*$ is computed using the SSE from the model in equation (3)

Let R be the likelihood ratio L_1/L_2 If the smaller model is true, then $Q = -2 \ln R$ which can be conveniently expressed as the difference $(2T \ln \hat{\sigma}_e^*) - (2T \ln \hat{\sigma}_e)$ is asymptotically distributed as χ_{DF}^2, with DF equal to the difference in the number of parameters in the two models This allows a comparison of the two models and, if the smaller model is inadequate, gives a measure of its inadequacy In particular, this gives a test whether $b_1 = b_2 = \cdots = b_B = 0$ and whether $d_1 = d_2 = \cdots = d_D = 0$

Note also that we assumed normally distributed errors in the construction of this test Although this provides a justification of the test, the asymptotic χ^2 distribution holds without normality, thus this procedure will still give valid results

Reference Notes

1 Stephenson, G Personal communication, November 1979

2 Stern, D Personal communication, June 1979

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$$\hat{s}_e^2 = \frac{1}{T-(A+B)} \sum_{t=1}^{T}\Big(M_t - \sum_{i=1}^{A} a_i M_{t-i} - \sum_{j=1}^{B} b_j B_{t-j}\Big)^2$$

$$= \frac{1}{T-(A+B)} \Sigma\,(\text{residuals})^2$$

$$= (SSE)/(T-(A+B))$$

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