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Detecting Cyclicity in Social Interaction

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This article reviews spectral and cross-spectral analytic methods for detecting cyclicity, cross-cyclicity, and lead–lag relationships in continuous data derived from the observation of dyadic interaction. It is found that lead–lag relationships can be assessed using the phase spectrum. Spectral analytic methods are then generalized to categorical observational data, and it is shown that by these methods one can derive the classical information theory definition of social communication and its distribution statistics.

Researchers who study social behavior are discovering that there are occasions when cyclical patterns characterize dyadic interaction, and thus they are searching for statistical techniques that can detect these cycles. The spectral analysis of time-series records was briefly suggested by Luce (1970) as a useful technique for the study of biological rhythms such as heart rate, respiration, REM sleep, and other cyclic biochemical and physiological processes. However, spectral analysis is not widely known to behavioral scientists, and it has yet to be used in the study of social interaction. A recent exception is the work of Hayes and Cobb (Note 1), who observed couples living in a laboratory setting, analyzed cycles of talk and silence using spectral analysis of time-series records, and related an observed cycle to human circadian rhythms.

Researchers who study dyadic social interaction are also interested in the bivariate case in which two time-series records are obtained, one from each of the two interacting organisms; the research question often involves the search for cycles in cross-correlations between the two series. For example, Kendon (1967) reported that when two people converse, the cycles of gaze and gaze aversion interlace, much as do sine and cosine waves. People are out of phase in eye-to-eye contact as a function of who is speaking; in particular, when a person begins speaking he or she looks away from the listener and begins increasing eye-to-eye contact time toward the end of the speech, which acts as an implicit signal for the listener to begin looking away and speaking.

Another example of cross-cyclicity is the work of Brazelton and his associates (e.g., Brazelton, Koslowski, & Main, 1974). Tronick, Als, and Brazelton (1977) studied mother–infant interaction and reported that the infants looked away following periods of maximum involvement with the mother and after a rest period became engaged again. Tronick et al. calculated synchrony and dissynchrony as running correlations between scaled scores of involvement, from maximum positive involvement to maximum negative involvement, but did not employ cross-spectral time-series methods. Cross-spectral analysis would have been an appropriate technique for studying both synchrony and lead–lag relationships between two time series in the Tronick et al. study.

Cross-spectral analysis may have considerable promise for studying interacting physiological systems within an organism. For example, Porges and his associates (Porges, Bohrer, Keren, Cheung, & Franks, Note 2) are using cross-spectral methods to study the linkage between respiration and heart rate. A function called *coherence* obtained from cross-spectral analysis is the equivalent of the

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square of the correlation between the two physiological systems as a function of their relative lag. Porges et al. found that the coherence between respiration and heart rate is related to cognitive attentional processes. Hyperactive children had low coherence between respiration and heart rate; low doses of methylphenidate had positive influence on cognitive performance and social behavior, whereas higher doses often resulted in lethargy. Porges and his associates are testing the model that deficits in linkage between the respiratory system and the cardiovascular systems are related to the attentional problems of hyperactive children and that low doses of methylphenidate mediate to increase the coherence between systems, thereby affecting cognitive functioning.

Because time-series techniques are not widely known to psychologists, this article reviews the spectral and cross-spectral analysis of continuous data. The present research also derives the new result that the slope of the phase spectrum of any two stationary processes can be used to detect lead–lag relationships. Lead–lag relationships are useful in making inferences about which series is, in some sense, driving the other. One application of lead–lag relationships is a redefinition of the concept of dominance in social interaction as an asymmetry in predictability in the time domain (Gottman, in press). This definition of dominance using cross-spectral analysis subsumes a range of observations about dominance across species. For example, the beta male in a group of monkeys is more responsive to the behavior of the alpha male than conversely (Maslow, 1936); that is, the behavior of the beta male is more predictable from past behavior of the alpha male than conversely.

Most researchers of social interaction collect categorical rather than continuous observational data (e.g., Hutt & Hutt, 1970; Lewis & Rosenblum, 1974). There are currently no statistical techniques for detecting cycles in one sequence and cycles between two sequences for categorical data over time. Categorical data collected over time can always be transformed to continuous time-series data; for example, for every block of k time units the local probability of each category can be computed, which produces a continuous variable for each category. For a discussion of categorical data types in observational research, see Gottman and Bakeman (in press).

In this article, I derive extensions of spectral time-series methods to categorical data. One result of these extensions is the derivation of the commonly used information theory definition of communication, summarized by Wilson (1975) as follows:

> Communication has been defined as the process by which behavior of one individual alters the probability of behavioral acts in other individuals . . . . In words, the conditional probability that act X_2 will be performed by individual B given that A performed X_1 is not equal to the probability that B will perform X_2 in the absence of X_1. (p. 194)

This is an important definition for the study of sequences in social interaction because it suggests the notion that a behavior in one organism has social communicative value to the extent that it reduces uncertainty in predicting the behavior of another organism. This definition is now widely used to detect sequences in dyadic interaction (for reviews, see Gottman & Bakeman, in press; Gottman & Notarius, 1978; Sackett, 1977).

Another result of the extension of spectral time-series methods to categorical data in this article is the demonstration of the validity (and limitations) of a statistical test of significance between conditional and unconditional probabilities recently suggested by Sackett (1977). After the information theory definition of communication is derived, spectral and cross-spectral methods are used to suggest how lead–lag relationships and cycles can be detected in categorical time series.

The Continuous Case

Granger and Hatanaka (1964) noted that the first time series subjected to spectral analysis were those that had a cycle with one dominant frequency, such as the 11-year oscillation in sunspot data and the annual cycle in meteorological data. They wrote,

> It was felt that if one could determine the amplitude period and phase of a sine curve sufficiently accurately and subtract this from the data, then the remainder ought to be an independent, random series. When, in fact, this was done and the remainder was still found to be somewhat too smooth, it was natural to re-use the current predominant idea of the cause of the

smoothness and to look for yet further sine curves to fit to the data. (pp. 4–5)

The model for a time series, X_t, was therefore a weighted sum of sine and cosine curves with an uncorrelated random remainder; if the number of observations, $n = 2q + 1$, is odd, one can write

$$X_t = A_0 + \sum_{i=1}^{q} (A_i \cos 2\pi f_i t + B_i \sin 2\pi f_i t) + \epsilon_t, \quad (1)$$

where $f_i = i/n$ is the ith harmonic of the fundamental frequency $1/n$. Fourier analysis makes it possible to derive least squares estimates for the coefficients:

$$\hat{A}_0 = \bar{X} = \frac{1}{n} \sum X_t;$$

$$\hat{A}_i = \frac{2}{n} \sum_{t=1}^{n} X_t \cos 2\pi f_i t;$$

$$\hat{B}_i = \frac{2}{n} \sum_{t=1}^{n} X_t \sin 2\pi f_i t.$$

This decomposition of a time series into component frequencies met with some initial success. For example, Whittaker and Robinson (1924) showed that the brightness of a variable star could be decomposed into two component frequencies, and they thus determined that the variable star was a binary star.

It would be useful to have some function that peaked at frequency bands that made major contributions to the variance of the series. For an infinite number of observations, the variance of the series at each frequency, f_i, is called the spectral density function, f. For a sample of n points it is called the *periodgram:* $I(f_i) = (1/8\pi)(A_i^2 + B_i^2)$. Because the sine and cosine terms in Equation 1 form an orthogonal set of functions, it can be shown that the variance of the time series is partitioned into independent parts by the periodogram:

$$\frac{1}{n} \sum (X_t - \bar{X})^2 = \frac{1}{2n} \sum I(f_i).$$

Early work on the spectral analysis of time series suggested that the periodogram was precisely the function that would peak at frequencies that contributed major portions to the variance of the time series; in fact, Schuster (1898) suggested that the periodogram be calculated and that its peaks be used to detect cycles. Subsequently, problems with spurious peaks led to the construction of significance tests for the periodogram (for a review of these tests, see Jenkins & Priestley, 1957). However, these tests were not adequate because the periodogram has some very poor statistical properties.

If the sample autocovariance at lag k is defined as

$$C_k = \frac{1}{n} \sum_{t=1}^{n-k} X_t X_{t-k},$$

then C_k is an unbiased estimator of the population autocovariance (Hannan, 1967), and it can be shown (Box & Jenkins, 1970, p. 45) that the periodogram is given by

$$I(f) = \frac{1}{2\pi}(C_0 + 2 \sum_{k=1}^{n-1} C_k \cos 2\pi f k),$$

where $0 \leq f \leq \frac{1}{2}$, which expresses that the periodogram is the Fourier transform of the sample autocovariance function. This implies that the periodogram is also easily calculated from the sample autocovariances; thus at first the problems of spectral time-series analysis appeared to be solved.

Unfortunately, although the periodogram does converge to the spectral density function, f, it does not converge uniformly; that is, its variance around f does not decrease to zero as n, the number of observations, increases (Hannan, 1967, pp. 52–53). In fact, Bartlett (1948) showed that the limit of the variance of the periodogram as n increases is $\sigma^4 f^2$, where σ^2 is the variance of the series. The failure of the periodogram led Tukey (1967) to make the following reflection:

If we dealt with problems involving the superposition of a few simple periodic phenomena, as do astronomers interested in binary stars and related problems, we can learn much from the periodogram. Sadly, however, almost no one else has this kind of data. As a result the periodogram has been one of the most misleading devices I know. (p. 25)

A dramatic illustration of Tukey's point is the periodogram of a series of random numbers, called white noise. White noise, like white light, is composed of all frequencies with equal intensities, and therefore its periodogram should be a straight line. Jenkins and Watts (1968) showed that the periodogram of white noise is not only not a straight line but continues to oscillate wildly as the number of observations is increased. However, the spurious peaks of the periodograms of each sample of white noise occur in random places on the frequency domain, and this provides the key to solving the problems of the periodogram. The average of many periodograms obtained from many samples of the same white noise process in fact tends toward a straight line as the number of observations in each sample increases.

This observation led Bartlett (1948) to suggest that the time series can be segmented and that a periodogram can be averaged across all segments. Bartlett showed that the averaged periodogram would coverage uniformly to the spectral density. Jenkins (1967) demonstrated that Bartlett's suggestion is equivalent to estimates of the form

$$f(f_i) = \frac{1}{2\pi}[C_0 + 2\sum_{j=1}^{n-1} \lambda_j(f_i)C_j \cos 2\pi f_i j]. \quad (2)$$

The function of $\lambda(f_i)$ is called a *spectral window*, and it weights the autocovariance function to ensure uniform convergence. Parzen's (1967) result is important because to implement Bartlett's suggestion would require an extremely long time series, whereas Jenkins's suggestion can be implemented with shorter time series, assuming that the window weighting function is suitably chosen. The most commonly used spectral window is the Tukey–Hanning window (Blackman & Tukey, 1958): $\lambda_j = 1 + \cos(\pi j/m)$, where m is an arbitrary integer, usually chosen so that $m < n/3$. (See Parzen, 1967, for a discussion of various spectral windows.) Thus a weighted Fourier transform of the autocovariance function does converge uniformly to the spectral density. Jenkins and Watts (1968) showed that the distribution of the intensity estimates at each frequency of the periodogram "will be very nearly a χ_2^2 regardless of the distribution of the [time-series] process" (p. 233). For the Tukey–Hanning window, the equivalent degrees of freedom must be modified (Granger & Hatanaka, 1964, pp. 59–64; Jenkins & Watts, 1968, pp. 248–257). In this article the term *spectrum* refers to the weighted periodogram.

An illustration of the spectrum may clarify its relationship as the Fourier transform (with an appropriate spectral window) of the autocovariance function. If the time series is a second-order autoregressive process,

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \epsilon_t, \quad (3)$$

where ϵ_t is an uncorrelated, random series and $\phi_1^2 + 4\phi_2 < 0$, then the behavior of the series will appear periodic. The constraints on ϕ_1 and ϕ_2 occur because periodicity only occurs when the roots of the characteristic equation of the process are imaginary (Box & Jenkins, 1970, p. 59). Note that this time series will not be deterministically periodic, as is a sine wave; there is a random component to the periodicity. In this case the autocovariance function will be a single-frequency damped sine wave.[1] Figure 1 is a plot of the autocorrelation function and spectrum of a simulated second-order autoregressive model. The spectrum shows only one peak[2]; a fourth-order autoregressive process is capable of representing a process with two peaks, and so on.

The relationship between the autocorrelation and spectrum of the process represented by Equation 3 is intuitively clear. If the time series is periodic, the autocorrelation should increase at multiples of the period. For

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[1] The expression for the theoretical autocorrelation function is

$$\rho_k = \frac{[\text{sgn}(\phi_1)]^k d^k \sin(2\pi f_0 k + F)}{\sin F},$$

where sgn $= +1$ if ϕ_1 is positive and sgn $= -1$ if ϕ_1 is negative. The factor d is called the *damping factor*, f_0 is called the *frequency*, and F is called the *phase*. These factors are related to the model parameters as follows:

$$d = [(-\phi_2)^{\frac{1}{2}} \text{sgn}(\phi_1)];$$

$$\cos 2\pi f_0 = \frac{|\phi_1|}{2(-\phi_2)^{\frac{1}{2}}};$$

$$\tan F = \frac{1 + d^2}{1 - d^2} \tan 2\pi f_0.$$

[2] The spectrum of a second-order autoregressive process can be written in closed form as

$$p(f) = 2\sigma_a^2/[1 + \phi_1^2 + \phi_2^2 - 2\phi_1(1 - \phi_2)\cos 2\pi f - 2\phi_2 \cos 4\pi f],$$

where $0 \leq f \leq \frac{1}{2}$. The spectrum reflects the periodic behavior of the second-order autoregressive process when the roots of its characteristic equation are complex.

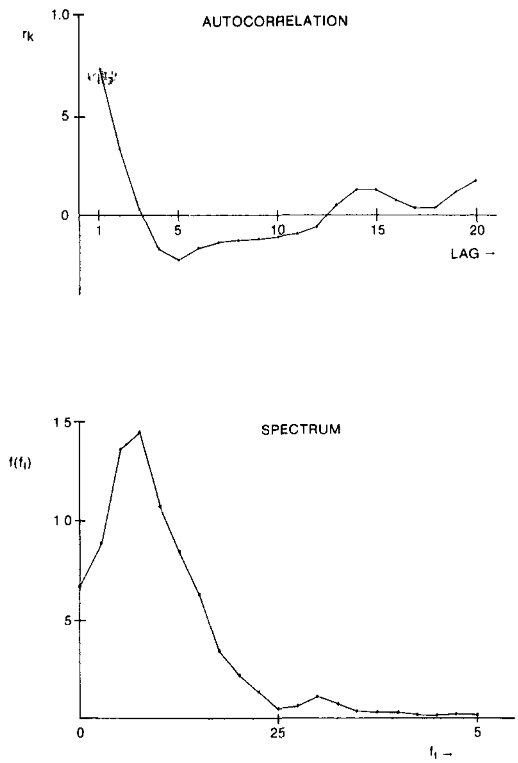

Figure 1. Autocorrelation function (r_k is the autocorrelation at lag k) and spectrum, $f(f_i)$, of one realization of a second-order pseudoperiodic time series. ($X_t = 1.1X_{t-1} - .5X_{t-2} + e_t$.)

example, for monthly wholesale wheat prices, the correlation between months 12 months apart (June with June, July with July, etc.) should be higher than that between months in different seasons. This relationship should fall off across years, and so the autocorrelation should resemble the damped sine wave in Figure 1. Since there is only one 12-month cycle, one would expect the spectrum (the weighted Fourier transform of the autocovariance function) to show only one peak. If the series had two cycles, the autocorrelation function would appear similar in shape (but more complex), and the spectrum would have two peaks.

Note that one cannot reconstruct the original time series simply by knowledge of the spectrum. This is true because very different series can be produced simply by adjusting the relative phases of the component frequencies; the phase of a sine wave determines its amplitude at time zero. Phase is a particularly important concept in the bivariate case.

For two time series, the generalization is not difficult. In fact, the Fourier transform (with suitable window) of the cross-covariance between the two time series is the cross-spectrum. The cross-spectrum has several components, a phase spectrum, and a cross-amplitude spectrum. The phase spectrum indicates

> whether the frequency components of one series lead or lag the same frequency components in the other series, and the cross-amplitude spectrum shows whether the amplitude of the component at a particular frequency in one series is associated with a large or small amplitude at the same frequency in the other series. (Jenkins & Watts, 1968, pp. 342–343)

The coherence is a function similar to the square of a correlation coefficient and is defined as the ratio of the square of the cross-spectrum divided by the product of the spectra of the individual series[3]; for two series, X_t and Y_t,

$$K_{xy}(f_i) = \frac{|f_{xy}(f_i)|^2}{f_{xx}(f_i)f_{yy}(f_i)}. \quad (4)$$

Distribution properties of these functions are discussed in Jenkins and Watts (1968, chap. 9); the properties for these functions with the Tukey–Hanning window are discussed in Granger and Hatanaka (1964, chap. 5). A coherence of one means that prediction is perfect from one series to another for all frequencies; a coherence of zero means that it is impossible to predict one series from the other. The prediction is of amplitude covariations in the two series, with no indication of lead–lag relationships, so that a complete description of relationships requires the phase spectrum as well as the coherence. If the coherence has one major peak, then the bulk of the correlation between the two processes is confined to a particular frequency band. If it is essential to predict correlations at major frequency bands of series Y_t, the coherence can be investigated at frequencies that have peaks in the spectrum of Y_t. An

[3] An alternative approach for specifying the relationship between two time series in the time domain, as opposed to in the frequency domain, is called *transfer function analysis* and is discussed by Box and Jenkins (1970).

alternative, suggested by Porges et al. (Note 2), is to compute one statistic called the *weighted coherence*, which is an estimate of the amount of variation in one series that can be accounted for by variation in the other:

$$\sum_i k_{xy}(f_i) f_{xx}(f_i) / \sum_i f_{xx}(f_i).$$

They wrote,

> Conceptually the coherence may be thought of as a time-series analogue of the omega-squared . . . . or the amount of variance accounted for by the influence of one series on the other. Therefore, the coherence times the spectral density estimate of heart rate activity at each frequency . . . . would describe the amount of heart rate activity which could be accounted for by respiration, i.e., the shared variance of heart rate and respiration. (p. 5)

If the cross-covariance is $C_{xy}(t)$, the unweighted cross-spectrum is the Fourier transform of the cross-covariance:

$$f_{xy}(f) = \frac{1}{2\pi} \sum_{-\infty}^{\infty} e^{i2\pi f t} C_{xy}(t), \quad \text{where } i = (-1)^{\frac{1}{2}}.$$

This complex number can be written as a real part plus an imaginary part: $f_{xy}(f) = C + iQ$. The phase spectrum is defined as

$$\phi_{xy}(f) = \arctan \frac{Q}{C}; \qquad (5)$$

C is called the cospectrum and Q the quadrature spectrum, and they measure the covariance between in-phase and out-of-phase components, respectively.

The slope of the phase spectrum determines the time-lag and the lead–lag relationships between the two series. For example, if one time series, $X(t) = \epsilon(t)$, is white noise with variance σ^2 and the other series is $Y(t) = X(t + L)$, then L is the *lead time* and Y leads X by L time units later. Since $X(t)$ is white noise, the covariance of $X(t)$ and $Y(t)$ is

$$\begin{aligned} C_{xy}(t) &= E[X(s)Y(s + t)] \\ &= E[X(s)X(s + t + L)] \\ &= \begin{cases} \sigma^2 & \text{at } t = -L \\ 0 & \text{otherwise} \end{cases} . \end{aligned}$$

The cross-spectrum is the Fourier transform of the cross-covariance. Assuming $\sigma = 1$, this

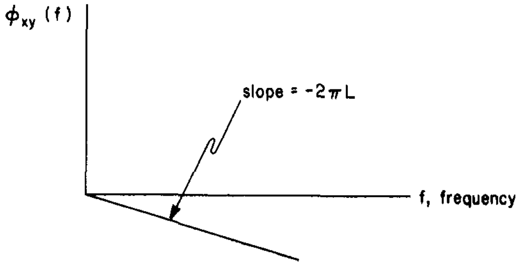

Figure 2. Phase spectrum, $\Phi_{xy}(f)$, when $Y(t)$ leads $X(t)$ by a constant time, L.

gives

$$\begin{aligned} f_{xy}(f) &= \frac{1}{2\pi} \sum_{-\infty}^{\infty} e^{i2\pi f t} C_{xy}(t) \\ &= \frac{\sigma^2}{2\pi} e^{-i2\pi f L} = \frac{\sigma^2}{2\pi} (\cos 2\pi f L - i \sin 2\pi f L), \end{aligned}$$

where $i = (-1)^{\frac{1}{2}}$. The phase spectrum is given by

$$\begin{aligned} \phi_{xy}(f) &= \arctan\left(\frac{Q}{C}\right) = \arctan\left(\frac{-\sin 2\pi f L}{\cos 2\pi f L}\right) \\ &= \arctan(-\tan 2\pi f L) = -2\pi f L. \end{aligned}$$

Therefore the phase spectrum will be a straight line that passes through the origin with negative slope proportional to the time lag, L (see Figure 2).

More generally, lead–lag relationships can be estimated by testing the significance of the slope of the least squares linear regression approximation to the phase spectrum.[4] It is important to note that this method does not give complete information; X and Y may be periodic at a particular frequency and have a constant phase relationship at that frequency

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[4] The phase spectrum shown in Figure 2 can be shown to hold for any two stationary processes that differ by a constant time lag. If $Y(t) = X(t + L)$, then the Fourier transform of $Y(t)$ is

$$F[Y(t)] = \int_{-\pi}^{\pi} Y(t) e^{iwt} dt = \int_{-\pi}^{\pi} e^{iwt} X(t + L) dt.$$

If one lets $u = t + L$, then

$$F[Y(t)] = \int_{-\pi}^{\pi} e^{iw(u-L)} X(u) du = e^{-iwL} \int_{-\pi}^{\pi} e^{iwu} X(u) du;$$

$$F[Y(t)] = e^{-iwL} F[X(t)].$$

Hence the Fourier transform of $Y(t)$ is the Fourier transform of $X(t)$ multiplied by the phase shift $e^{-i\phi}$, where $\phi = -wL$.

but some other phase relationship at another frequency. The slope of the phase spectrum averages the lead–lag relationship across all frequencies, and it may be important in a particular investigation to determine the phase relationship between X and Y at specific frequencies of interest. One alternative discussed by Granger and Hatanaka (1964) is a two-component model in which the frequency domain is divided in half and lead–lag relationships are assessed separately for slow and rapid components. For these calculations, computer programs are available in most universities that have the University of California, Los Angeles biomedical series (Dixon, 1974, pp. 517–582, Programs 2T, 3T, and 4T).

The Categorical Case

In the categorical case two series, X_t and Y_t, are set equal to one if the characteristics that they represent are observed and equal to zero otherwise. The unbiased estimator of the cross-covariance, lagged k units in time is

$$C_{xy}(k) = \frac{1}{n-k}\sum_{1}^{n-k}(X_t - \bar{X})(Y_{t+k} - \bar{Y}).$$

For categorical data, $\bar{X}$ and $\bar{Y}$ are the unconditional probabilities, p_x and p_y, that X_t and Y_t are one in $n-k$ observations, so that

$$\begin{aligned} C_{xy}(k) &= \frac{1}{n-k}\sum_{1}^{n-k}(X_t - p_x)(Y_{t+k} - p_y) \\ &= \frac{1}{n-k}\Big[\sum_{1}^{n-k} X_t Y_{t+k} - p_y\cdot\sum_{1}^{n-k} X_t \\ &\quad - p_x\cdot\sum_{1}^{n-k} Y_{t+k} + p_x p_y (n-k)\Big] \\ &= \frac{1}{n-k}\Big[\sum_{1}^{n-k} X_t Y_{t+k} - p_y p_x (n-k) \\ &\quad - p_x p_y (n-k) + p_x p_y (n-k)\Big]; \\ C_{xy}(k) &= \frac{1}{n-k}\Big[\sum_{1}^{n-k} X_t Y_{t+k} \\ &\quad - p_y p_x (n-k)\Big]. \qquad (6) \end{aligned}$$

The sum in Equation 6 is simply the number of lagged-k (1, 1) pairs. Note that by definition, the conditional probability that Y is equal to one given that X was equal to one k time units ago, $p_k(Y|X)$, is simply the number of (1, 1) pairs at lag k divided by the number of occurrences of $X = 1$ in $n-k$ observations. If one denotes the number of (1, 1) pairs at lag k as $M_{xy}(k)$, then, from the definition of the lagged conditional probability, it follows that $p_k(Y|X) = M_{xy}(k)/p_x(n-k)$. Therefore, the number of (1, 1) pairs at lag k is

$$M_{xy}(k) = \sum_{1}^{n-k} X_t Y_{t+k} = p_k(Y|X)(p_x)(n-k).$$

Substituting this back into Equation 6 gives

$$C_{xy}(k) = p_x[p_k(Y|X) - p_y], \qquad (7)$$

as the categorical equivalent of the cross-covariance.

This function is proportional to the information theory definition of communication assessed as the difference between conditional and unconditional probabilities.

To derive the distribution of the covariance, the variance of the covariance can be computed as follows:

$$\begin{aligned} C_{xy}(k) &= p_x[p_k(Y|X) - p_y]; \\ C_{xy}(k) - \bar{C} &= p_x p_k(Y|X) - p_x\overline{p_k(Y|X)} \\ &= p_x[p_k(Y|X) - \overline{p_k(Y|X)}]; \\ \text{var}\,[C_{xy}(k)] &= p_x^2\{\text{var}\,[p_k(Y|X)]\}. \end{aligned}$$

Under the null hypothesis of no relationship between the two categorical time series, X_t and Y_t, $p_k(Y|X) = p_y$, and the variance of the unconditional probability of a dichotomous variable that is not autocorrelated is $p_y(1-p_y)/m$ (Siegel, 1956, p. 40), where m = the number of observations used to calculate p_y. For the covariance $C_{xy}(k)$, $m = n - k$, and the result is

$$\text{var}\,[C_{xy}(k)] = p_x^2 p_y(1-p_y)/(n-k).$$

Since under the null hypothesis, $C_{xy}(k)/SD[C_{xy}(k)]$ is normally distributed with mean zero and unit variance ($N[0, 1]$) (Box & Jenkins, 1970), one has

$$\frac{C_{xy}(k)}{SD[C_{xy}(k)]} = \frac{p_x[p_k(y|x) - p_y]}{[p_x^2 p_y(1-p_y)/(n-k)]^{\frac{1}{2}}} \sim N(0, 1);$$

$$Z = \frac{p_k(y|x) - p_y}{[p_y(1-p_y)/(n-k)]^{\frac{1}{2}}} \sim N(0, 1). \qquad (8)$$

This is a derivation of a statistic that was recently proposed by Sackett (1977).

An estimate of the error introduced in Equation 8 by autocorrelation in each series, under the null hypothesis of no cross-correlation, can be obtained by using the expression for the variance of the cross-correlation under the null hypothesis given by Box and Jenkins (1970, p. 377):

$$\operatorname{var}[r_{xy}(k)] \simeq \frac{1}{n-k}[1 + \sum_{j=1}^{\infty} r_{xx}(j) r_{yy}(j)] \simeq \frac{1}{n-k}(1+\delta).$$

Thus, the variance of the cross-correlation would be $1/(n-k)$ if there were no autocorrelation. To estimate the quantity δ, rewrite the autocorrelations using

$$r_{xx}(k) = C_{xx}(k)/C_{xx}(0):$$

$$\delta = \sum_{1}^{\infty} r_{xx}(j) r_{yy}(j) = \frac{1}{C_{xx}(0) C_{yy}(0)} \sum_{1}^{\infty} C_{xx}(j) C_{yy}(j).$$

Now substitute the quantity for the covariance from Equation 7:

$$\delta = \frac{p_x p_y}{p_x(1-p_x) p_y(1-p_y)} \sum_{1}^{\infty} [p_j(x|x) - p_x] \times [p_j(y|y) - p_y].$$

If one assumes that the quantity in the sum decreases exponentially with increasing lag and one denotes

$$\theta = [p_1(x|x) - p_x][p_1(y|y) - p_y],$$

then

$$\delta = \frac{1}{(1-p_x)(1-p_y)} \cdot \frac{\theta}{(1-\theta)}.$$

Delta is a maximum when the conditionals are one and a minimum when the conditionals equal the unconditionals:

$$\delta_{\max} = \frac{1}{1-(1-p_x)(1-p_y)}; \quad \delta_{\min} = 0.$$

The cross-spectral density function for categorical data can be written as the Fourier transform of the cross-covariance (weighted by a suitable window), and this function will behave in a fashion similar to the continuous case. The generalizations are obtained by applying Equation 2 to Equation 7: The cross-spectrum is

$$f_{xy}(f_i) = \frac{1}{2\pi}[C_{xy}(0)\lambda_0(f_i) + 2 \sum_{j=1}^{n-1} \lambda_j(f_i) C_{xy}(j) \cos 2\pi f_i j].$$

The spectrum of X_t is

$$f_{xx}(f_i) = \frac{1}{2\pi}[p_x(1-p_x)\lambda_0(f_i) + 2 \sum_{j=1}^{n-1} \lambda_j(f_i) C_{xx}(j) \cos 2\pi f_i j].$$

The spectrum of Y_t is

$$f_{yy}(f_i) = \frac{1}{2\pi}[p_y(1-p_y)\lambda_0(f_i) + 2 \sum_{j=1}^{n-1} \lambda_j(f_i) C_{yy}(j) \cos 2\pi f_i j].$$

The lambdas are the Tukey–Hanning weights (Blackman & Tukey, 1958).

To summarize, Equation 7 is the categorical equivalent of the cross-correlation, and if $X = Y$, of the autocorrelation. If cyclicity exists in a series of categorical data with one major cycle, then $C_{xy}(k)$ should behave as a damped sine wave of Figure 1, and the spectrum should show one peak. An examination of the spectrum, which is the weighted Fourier transform of Equation 7, reveals major cycles in the categorical series. The coherence and phase spectrum are similarly generalized, and the slope of the phase spectrum detects lead–lag relationships that span all component frequencies. Computationally, all these statistics can be calculated simply by inputting each series as a binary zero–one time series.

To illustrate the relationship between continuous and dichotomous spectral time-series statistics, one example that compares statistics for continuous data and the same data dichotomized around the mean is presented.

Example

The data in this example are derived from coding a videotape of a married couple working

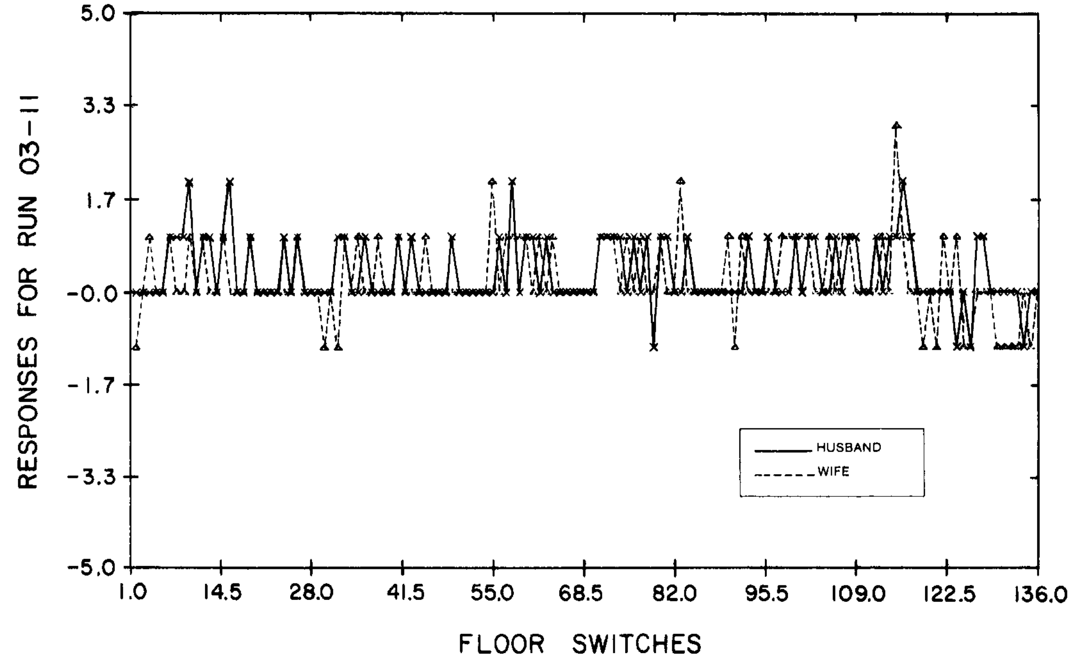

Figure 3. Positivity of behaviors of one couple on an improvised conflict task.

on an improvised conflict task. The coding system and the method for generating the time series from categorical data are described in Gottman, Markman, and Notarius (1977). The graphs displayed in Figure 3 represent a tally of positive minus negative nonverbal behavior coded from voice tone, facial expressions, and body cues. The unit plotted on the abscissa is the "floor switch," that is, the set of utterances before one person gives up the floor to the other.

These data were transformed to categorical data by dichotomizing around the mean of each series, and phase spectra and the coherences were calculated for both the discrete and the continuous cases using Tukey–Hanning

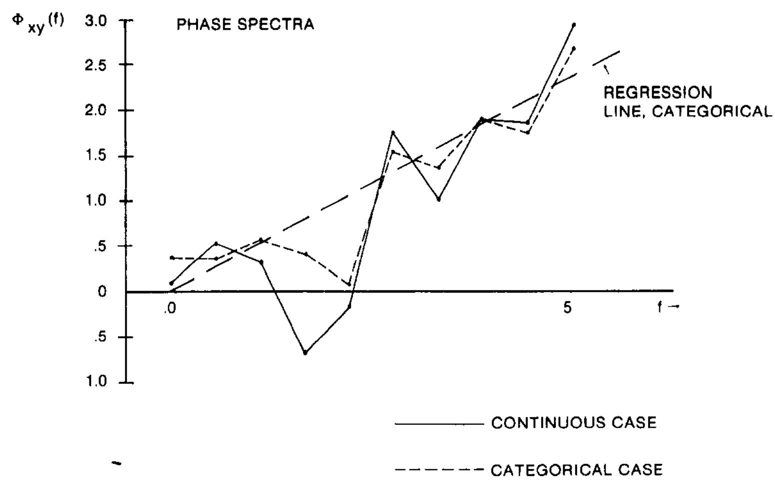

Figure 4. Phase spectra for continuous and dichotomous case of couple in Figure 3; $\Phi_{xy}(f)$ = phase spectrum; f = frequency.

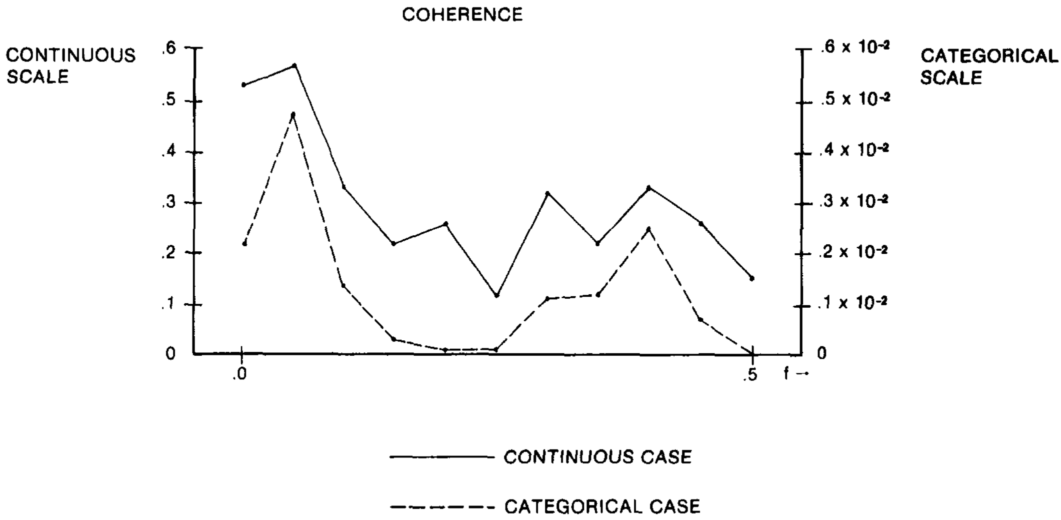

Figure 5. Coherence spectra for continuous and dichotomous cases of couple in Figure 3; f = frequency.

weights and the Fast-Fourier transform program available at the University of Illinois (SOUPAC programs). Figures 4 (see Equation 5) and 5 (see Equation 4) present a comparison of these two statistics for the continuous and categorical cases. The phase spectra are nearly identical and have very similar regression lines that in both cases are interpreted as the wife leading the husband, with a constant lag equal to the slope of the regression line. The slope is .31 for the continuous case and .25 for the categorical case.

The coherence for the categorical case is much lower, which is not surprising because so much information about strength of association is lost by dichotomizing. However, the important aspect of the coherence is the location of peaks, and one can see that the coherence for the categorical case has a shape similar to that of the continuous case. The two highest peaks (at f = .1 and f = .4) are the same for both cases, so that information about cyclicity in the strength of association across series is preserved.

Conclusion

Spectral and cross-spectral time-series methods were reviewed in this article for continuous data, and interpretations were discussed for the spectrum, the coherence, the weighted coherence, and the phase spectrum. These methods were also extended to categorical data. The extension made it possible to derive the information theory statistic for comparing conditional with lagged unconditional probabilities and for exploring the limits of the z-score test as a function of autocorrelation. Subsequent investigations should generate stochastic time-series data by using known autoregressive-moving average models with seasonal components (Box & Jenkins, 1970) and by comparing continuous and dichotomous analyses. The methods proposed in this article need to be applied to a range of problems, and their ability to describe patterns in data across time and to fail to detect patterns in known random data needs to be assessed empirically.

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